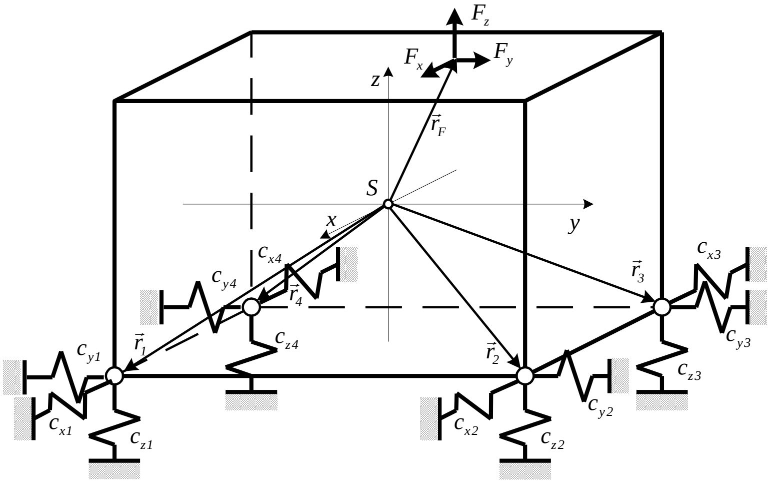
Dynamics of Machines Week 10 – Exercise

Vibration Analysis of 3D Model of a Machine and Machine Base



Equation of motion: $\ddot{a}_i + 2\xi_i\alpha_i\dot{a}_i + \alpha_i^2 a_i = Q_{ai}$ $\qquad (i = 1,2,3,4,5,6)$

Rearrangement: $\ddot{a}_i = -2\xi_i\alpha_i\dot{a}_i - \alpha_i^2 a_i + Q_{ai}$ $\qquad (i = 1,2,3,4,5,6)$

Mass of the machine and machine base is $m = 500kg$, non zero elements of the inertia tensor $J_{Sx} = 1900kgm^2$, $J_{Sy} = 1700kgm^2$, $J_{Sz} = 1500kgm^2$, spring constant values are the same at every vertices $c_{xi} = 5\cdot 10^{-5} m/N$, $c_{yi} = 5\cdot 10^{-5} m/N$, $c_{zi} = 2\cdot 10^{-5} m/N$, $(i = 1,2,3,4)$.

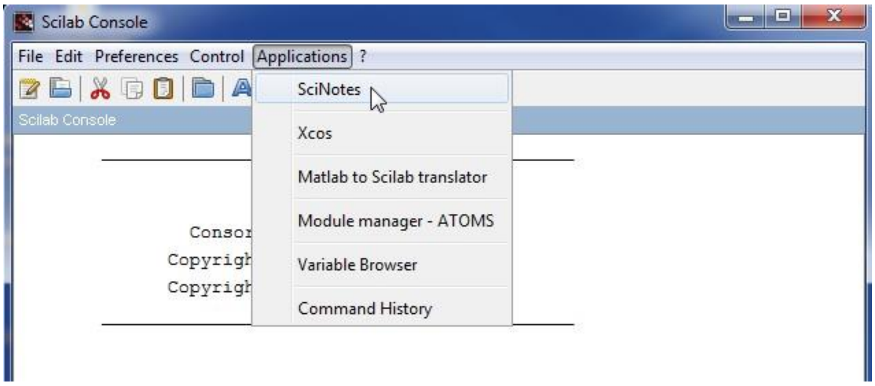
Coordinates of the vertices:
$(x_1 = 0{,}6m,\ y_1 = -0{,}7m,\ z_1 = -0{,}4m)$, $(x_2 = 0{,}6m,\ y_2 = 0{,}8m,\ z_2 = -0{,}4m)$,
$(x_3 = -0{,}5m,\ y_3 = 0{,}8m,\ z_3 = -0{,}4m)$, $(x_4 = -0{,}5m,\ y_4 = -0{,}7m,\ z_4 = -0{,}4m)$.

Excitation angular frequency: $\omega = 62{,}8\ (rad/s)$
Coordinates of excitation force: $F_z = 500\cos(62{,}8t)$
Coordinates of the position vector of excitation force, $\vec{r}_F$: $x_F = -0{,}3m,\ y_F = 0{,}2m,\ z_F = 0{,}4m$

Lehr damping coefficient in every natural frequency: $\xi = 0{,}1$

Tasks: Determine the displacement of point P as a function of time!
Position vector of point P: $\vec{r}_P = (0{,}5\vec{i} + 0{,}6\vec{j} + 0{,}4\vec{k})$ m
Final time $t_{max} = 15\ (s)$. The machine revs up from rest to the working engine speed during $t_{start} = 3(s)$. Length of a time step is $\Delta t = 0{,}001\ (s)$.

Solving method: The problem will be solved in two steps. In the first step the undamped natural frequencies (eigenvalues) and eigenvectors will be determined and in the second step the uncoupled (independent) damped differential equations will be solved with Runge-Kutta method.




```
// week 10 - exercise
clear;
usecanvas(%F);
//stacksize('max')

// Variables:
// mass (kg) and moment of inertia variables (kgm^2)
m=500; Jsx=1900; Jsy=1700; Jsz=1500;
// spring constants (m/N)
cxi=5.0e-5; cyi=5.0e-5; czi=2.0e-5;
// coordinates of the 4 vertices (m)
x1= 0.6; y1=-0.7; z1=-0.4;
x2= 0.6; y2= 0.8; z2=-0.4;
x3=-0.5; y3= 0.8; z3=-0.4;
x4=-0.5; y4=-0.7; z4=-0.4;
// Coordinates of the position vector of excitation force (m)
xF=-0.3; yF= 0.2; zF= 0.4;
// Coordinates of the position vector of point P
xP=+0.5; yP= 0.6; zP= 0.4;
// Lehr damping in every natural frequency
xsi=zeros(6,1);
xsi(1,1)=0.1;
xsi(2,1)=0.1;
xsi(3,1)=0.1;
xsi(4,1)=0.1;
xsi(5,1)=0.1;
xsi(6,1)=0.1;

F=zeros(3,1); F(3,1)= 500;          // coordinates of the force vector (N)
omega=62.8;                         // excitation angular frequency (rad/s)

// Initial conditions matrix (initial displacement and initial velocity values)
abi0(1,1)=0;abi0(2,1)=0;abi0(3,1)=0;abi0(4,1)=0;abi0(5,1)=0;abi0(6,1)=0;
abi0(7,1)=0;abi0(8,1)=0;abi0(9,1)=0;abi0(10,1)=0;abi0(11,1)=0;abi0(12,1)=0;
```

```
// time variables:
tmax=15;            // final time (s)
tstart=3;           // (s)
dt=0.001;           // time step (s)
t0=0;               // initial time (s)
t=0:dt:tmax;        // time interval (s)
// multiplier matrices of position vectors
R1=zeros(3,3);
R1(1,2)= z1;  R1(1,3)=-y1;
R1(2,1)=-z1;  R1(2,3)= x1;
R1(3,1)= y1;  R1(3,2)=-x1;
R2=zeros(3,3);
R2(1,2)= z2;  R2(1,3)=-y2;
R2(2,1)=-z2;  R2(2,3)= x2;
R2(3,1)= y2;  R2(3,2)=-x2;
R3=zeros(3,3);
R3(1,2)= z3;  R3(1,3)=-y3;
R3(2,1)=-z3;  R3(2,3)= x3;
R3(3,1)= y3;  R3(3,2)=-x3;
R4=zeros(3,3);
R4(1,2)= z4;  R4(1,3)=-y4;
R4(2,1)=-z4;  R4(2,3)= x4;
R4(3,1)= y4;  R4(3,2)=-x4;

RF=zeros(3,3);
RF(1,2)= zF;  RF(1,3)=-yF;
RF(2,1)=-zF;  RF(2,3)= xF;
RF(3,1)= yF;  RF(3,2)=-xF;


RP=zeros(3,3);
RP(1,2)= zP;  RP(1,3)=-yP;
RP(2,1)=-zP;  RP(2,3)= xP;
RP(3,1)= yP;  RP(3,2)=-xP;

// mass matrix
M=zeros(6,6);
M(1,1)=m;    M(2,2)=m;    M(3,3)=m;
M(4,4)=Jsx;  M(5,5)=Jsy;  M(6,6)=Jsz;
MM=M;

// spring matrix
C=zeros(6,6);
Ci=zeros(3,3);
Ci(1,1)=1/cxi;    Ci(2,2)=1/cyi;    Ci(3,3)=1/czi;
```

$$\underline{\underline{R}}_i = \begin{bmatrix} 0 & z_i & -y_i \\ -z_i & 0 & x_i \\ y_i & -x_i & 0 \end{bmatrix}$$

$$\underline{\underline{M}} = \begin{bmatrix} \underline{\underline{m}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{J}}_S \end{bmatrix} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & J_{Sx} & 0 & 0 \\ 0 & 0 & 0 & 0 & J_{Sy} & 0 \\ 0 & 0 & 0 & 0 & 0 & J_{Sz} \end{bmatrix}$$

$$\underline{\underline{C}}_i = \begin{bmatrix} \frac{1}{c_{xi}} & 0 & 0 \\ 0 & \frac{1}{c_{yi}} & 0 \\ 0 & 0 & \frac{1}{c_{zi}} \end{bmatrix}$$

```
////// stiffness matrix
C(1:3,1:3)=Ci+Ci+Ci+Ci;
C(1:3,4:6)=Ci*R1+Ci*R2+Ci*R3+Ci*R4;
C(4:6,1:3)=R1'*Ci+R2'*Ci+R3'*Ci+R4'*Ci;
C(4:6,4:6)=R1'*Ci*R1+R2'*Ci*R2+R3'*Ci*R3+R4'*Ci*R4;
CC=C;
```

$$\underline{\underline{C}} = \begin{bmatrix} \sum_{i=1}^{4} \underline{\underline{C}}_i & \sum_{i=1}^{4} \underline{\underline{C}}_i \underline{\underline{R}}_i \\ \sum_{i=1}^{4} \underline{\underline{R}}_i^T \underline{\underline{C}}_i & \sum_{i=1}^{4} \underline{\underline{R}}_i^T \underline{\underline{C}}_i \underline{\underline{R}}_i \end{bmatrix}$$

```
// generalized eigenvalue problem
[al,be,X]=spec(C,M);

alfa=zeros(6,1);
alfa2=zeros(6,1);
```

$$\underline{\underline{C}}\,\underline{q}_{0i} = \alpha_i^2 \underline{\underline{M}}\,\underline{q}_{0i} \qquad (i=1,2,...,6)$$

```
for i=1:6
    // eigenvectors normalization
    di2=X(:,i)'*MM*X(:,i);
    di=sqrt(di2);
    X(:,i)=X(:,i)/di;
    // natural frequency squares
    alfa2(i,1)=X(:,i)'*CC*X(:,i);
    // natural frequencies
    alfa(i,1)=sqrt(alfa2(i,1));
end
alfa_diffmethod=sqrt(al./be);    //natural frequencies with a different method
```

$$\underline{q}_{ni} = \frac{\underline{q}_{0i}}{\sqrt{\underline{q}_{0i}^T \underline{\underline{M}}\,\underline{q}_{0i}}} \qquad (i=1,2,...,6)$$

```
// real part of complex numbers
X=real(X);
alfa2=real(alfa2);
alfa=real(alfa)

// generalized force
QF0=zeros(6,1);
QF0(1:3,1)=F;
QF0(4:6,1)=RF'*F;
```

$$\underline{Q}_F = \begin{bmatrix} \underline{F} \\ \underline{\underline{R}}_F^T \underline{F} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \\ -z_F F_y + y_F F_z \\ z_F F_x - x_F F_z \\ -y_F F_x + x_F F_y \end{bmatrix}$$

```
// generalized force in the modal space
QF1=X'*QF0;
```

$$Q_{ai} = \underline{q}_{ni}^T \underline{Q}$$

```
function dabi=equation(t, abi)
    fact=1;
      if t < tstart then
        fact=t/tstart;
      end
    dabi(1)=abi(2);
    dabi(2)=-2*xsi(1,1)*alfa(1,1)*abi(2)-alfa2(1,1)*abi(1)+QF1(1,1)*cos(fact*omega*t);
    dabi(3)=abi(4);
    dabi(4)=-2*xsi(2,1)*alfa(2,1)*abi(4)-alfa2(2,1)*abi(3)+QF1(2,1)*cos(fact*omega*t);
```

$$\ddot{a}_i = -2\xi_i \alpha_i \dot{a}_i - \alpha_i^2 a_i + Q_{ai} \qquad (i=1,2,3,4,5,6)$$

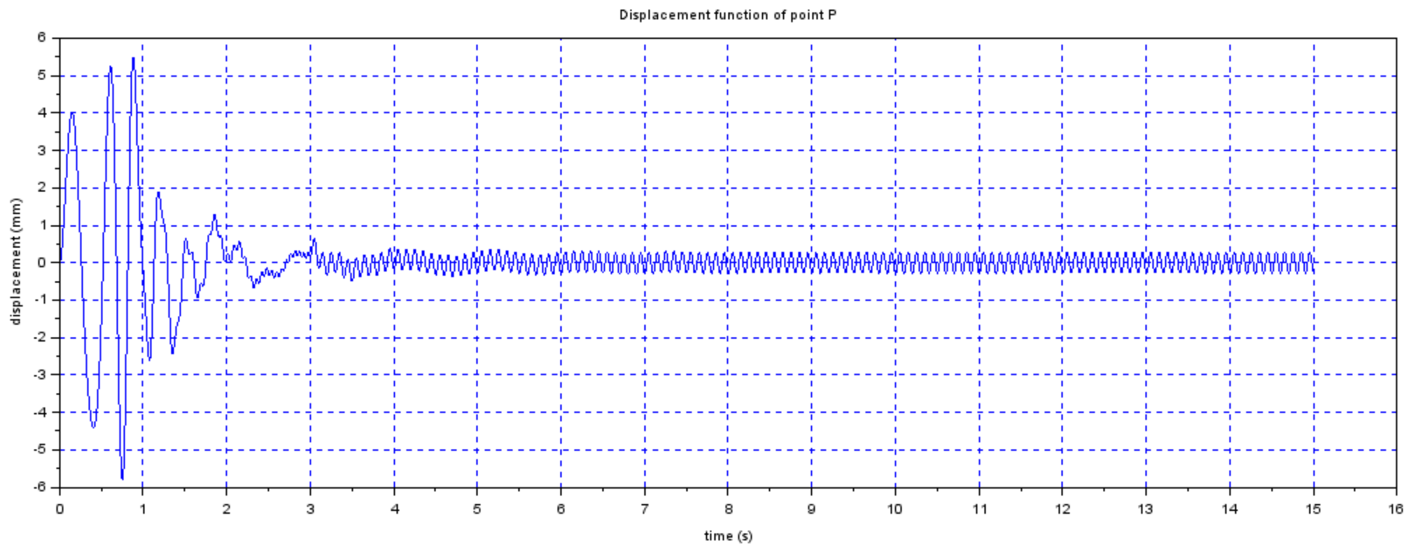
```
    dabi(5)=abi(6);
    dabi(6)=-2*xsi(3,1)*alfa(3,1)*abi(6)-alfa2(3,1)*abi(5)+QF1(3,1)*cos(fact*omega*t);
    dabi(7)=abi(8);
    dabi(8)=-2*xsi(4,1)*alfa(4,1)*abi(8)-alfa2(4,1)*abi(7)+QF1(4,1)*cos(fact*omega*t);
    dabi(9)=abi(10);
    dabi(10)=-2*xsi(5,1)*alfa(5,1)*abi(10)-alfa2(5,1)*abi(9)+QF1(5,1)*cos(fact*omega*t);
    dabi(11)=abi(12);
    dabi(12)=-2*xsi(6,1)*alfa(6,1)*abi(12)-alfa2(6,1)*abi(11)+QF1(6,1)*cos(fact*omega*t);
endfunction

// differential equation solving
abi=ode(abi0,t0,t,equation);

// sorting displacement and velocity
for i=1:6
  ai(i,:)=abi(2*i-1,:);        // normalized displacement
  bi(i,:)=abi(2*i,:);          // normalized velocity
end

// general displacement coordinates
qi=X*ai;
uvw=qi(1:3,:);
fixyz=qi(4:6,:);

// displacement of point P
uvwP=uvw+RP*fixyz;
```

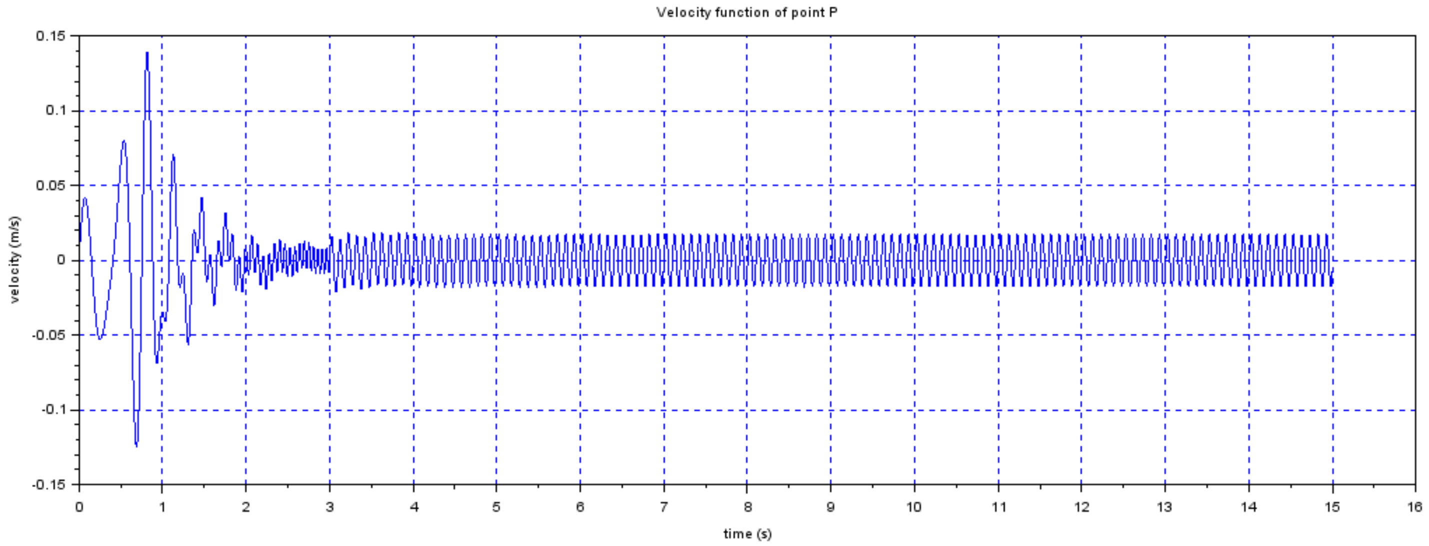
$$\underline{\underline{u}}_P = \underline{\underline{u}}_S + \underline{\underline{R}}_P \underline{\underline{\varphi}}$$

```
subplot(2,1,1);
plot(t,uvwP(3,:)*1000)
xtitle("Displacement function of point P","time (s)","displacement (mm)")
xgrid(2)


qdi=X*bi;
duvw=qdi(1:3,:);
dfixyz=qdi(4:6,:);

// velocity of point P
duvwP=duvw+RP*dfixyz;
```

$$\underline{\underline{\dot{u}}}_P = \underline{\underline{\dot{u}}}_S + \underline{\underline{R}}_P \underline{\underline{\dot{\varphi}}}$$

```
subplot(2,1,2);
plot(t,duvwP(3,:))
xtitle("Velocity function of point P ","time (s)","velocity (m/s)")
xgrid(2)
```


Displacement function of point P
displacement (mm)
time (s)


Velocity function of point P
velocity (m/s)
time (s)