SZÉCHENYI ISTVÁN UNIVERSITY | **DEPARTMENT OF APPLIED MECHANICS**

MECHANICS–DYNAMICS	**Theoretical questions and answers for BSc students**

(0) Mathematical bases
Theoretical questions may include simple numerical examples that require operations with vectors and tensors.

- dot product of vectors:
  Given: vector $\vec{a}$ and $\vec{b}$.
  Task: to be determined $\vec{a} \cdot \vec{b} = ...$
- cross product of vectors:
  Given: vector $\vec{a}$ and $\vec{b}$.
  Task: to be determined $\vec{a} \times \vec{b} = ...$
- vector triple product:
  Given: vector $\vec{a}$, $\vec{b}$, and $\vec{c}$.
  Task: to be determined $\vec{a} \times \left(\vec{b} \times \vec{c} \right) = ...$
- dot product of a tensor and a vector:
  Given: matrix of a tensor $\underline{\underline{T}}$ and a vector $\vec{a}$.
  Task: to be determined $\underline{\underline{T}}\vec{a} = ...$
- unit vector:
  Given: angle φ between axis x and a unit vector $\vec{e}$ lying in plane x, y.
  Task: to be determined $\vec{e} = ...$

(1) Define the concept of kinematics.

Kinematics deals with the description of motion but does not investigate the causes (forces) of motion.

(2) Define the concept of kinetics.

Kinetics investigates the causes of motion (the forces that cause motion). Its purpose is to determine motion knowing the causes.

(3) Define the concept of a point mass.

1st definition: geometrical point with material features.
2nd definition: a body whose position (motion) can be defined by the position (motion) of one of its points unambiguously.

(4) Define the concept of a rigid body.

It is body model in which the distance between any two points is constant. (The distance between points does not change under load.)

(5) Define the concept of the degree of freedom.

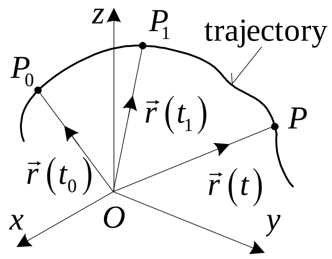
Degree of freedom is the number of linearly independent scalar data (coordinates) that determine the position of a body unambiguously in the space or on a plane.

(6) Define the position function of motion and trajectory (curve) of a point mass.

Position function is a scalar-to-vector function that defines the instantaneous position of the point mass.

1st definition: the trajectory curve is the spatial curve at which the point mass moves.

2nd definition: the trajectory curve is the spatial curve defined by the position function.



(7) Define the unit direction vectors of the natural coordinate system (Frenet–Serret frame) of a spatial curve.

$\vec{e}$ - tangent unit direction vector: $\vec{e} = \dfrac{d\vec{r}}{ds}$, $|\vec{e}| = 1$.

$\vec{n}$ - normal unit direction vector: $\vec{n} = \dfrac{d\vec{e}}{ds} = \kappa \vec{n} = \dfrac{1}{\rho}\vec{n}$, $|\vec{n}| = 1$.

$\vec{b}$ - binormal unit direction vector: $\vec{b} = \vec{e} \times \vec{n}$, $|\vec{b}| = 1$.

(8) Define the velocity function of a point mass. Give the definition of instantaneous velocity vector of a point mass. What are the properties of the instantaneous velocity vector?

The velocity function is the first derivative of the position function with respect to time:

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt}.$$

The instantaneous velocity is the value of the velocity function in a given moment: $\vec{v} = \vec{v}(t_1)$.

Properties:

- the instantaneous velocity is vector
- the direction of the instantaneous velocity vector is identical with the direction of the tangential unit direction vector of the trajectory

(9) What is the trajectory velocity of a point mass? What are its properties?

The trajectory velocity is the first derivative of the arc length (measured on the trajectory) with respect to time: $v(t)=\dfrac{ds(t)}{dt}$.

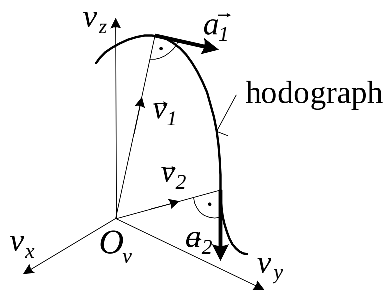
Properties:

- the trajectory velocity is the tangential coordinate of the velocity function
- the trajectory velocity is a scalar quantity (with sign)
- the sign of the trajectory velocity is determined by the dierction of the arc length *s*

(10) What is the hodograph? What is its most important feature?

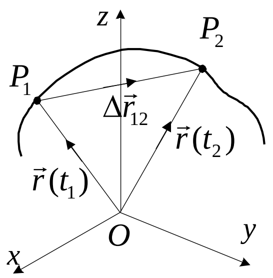
1st definition: it is a curve that is formed by the endpoints of the velocity vectors $\vec{v}(t)$ of the point mass in coordinate system $v_x,\ v_y,\ v_z$.

2nd definition: it is a curve defined by the endpoints of the velocity vectors when the velocity vectors are measured from a common starting point.



The most important feature is that the tangent vectors of a hodograph are the acceleration vectors.

(11) Define the average velocity of a point mass.



The average velocity for interval $\langle t_1,\ t_2\rangle$:

$$v_a=\frac{\vec{r}(t_2)-\vec{r}(t_1)}{t_2-t_1}=\frac{\vec{r}_2-\vec{r}_1}{t_2-t_1}=\frac{\Delta\vec{r}_{12}}{\Delta t_{12}}.$$

(12) Define the acceleration function of a point mass. Give the definition of instantaneous acceleration vector of a point mass. What are the properties of the instantaneous acceleration vector?

The acceleration function is the first derivative of the velocity function with respect to time and it is the second derivative of the position function with respect to time: $\vec{a}(t)=\dfrac{d\vec{v}(t)}{dt}=\dfrac{d^2\vec{r}(t)}{dt^2}$.

The instantaneous acceleration is the value of the acceleration function in a given moment: $\vec{a}=\vec{a}(t_1)$.

Properties:

- the instantaneous acceleration is vector
- the instantaneous acceleration vector lies in the osculating plane of the trajectory and it has components in the directions of tangent vector $\vec{e}$ and normal $\vec{n}$ vector: $\vec{a}=a_e\vec{e}+a_n\vec{n}$.

(13) Give the name, method of calculation and physical meaning of the coordinates of the acceleration vector of a point mass in the natural coordinate system $\vec{e}, \vec{n}, \vec{b}$.

Name	Method of calculation	Physical meaning
tangential acceleration	$a_e = \dfrac{dv(t)}{dt} = \dfrac{d^2 s(t)}{dt^2}$	It characterises the change of the magnitude of the velocity vector.
normal acceleration	$a_n = \dfrac{v^2}{\rho}$	It characterises the change of the direction of the velocity vector.

(14) What functions are called kinematical (phoronomical) functions and how are they related to each other?

Kinematical (phoronomical) functions are the following scalar-to-scalar functions:
$s = s(t),\ v = v(t),\ a_e = a_e(t)$.
Relations between the kinematical (phoronomical) functions:

Known: $s = s(t)$	Known: $a_e = a_e(t),\ s_0,\ v_0$
To be determined: $v(t) = \dfrac{ds(t)}{dt}$, $a_e = \dfrac{dv(t)}{dt} = \dfrac{d^2 s(t)}{dt^2}$.	To be determined: $v(t) = v_0 + \int_{t_0}^{t} a_e(t)\,dt$, $s(t) = s_0 + \int_{t_0}^{t} v(t)\,dt$.

(15) Define the (recti)linear and the planar motion of a point mass.

Rectilinear or linear or straight-line motion: the curvature of the trajectory of the point mass is zero ($\kappa = 0$), that is the radius of curvature goes to infinity at all points on the trajectory ($\rho \to \infty$).
Planar motion: if the point mass does not leave the plane spanned by the initial velocity vector $\vec{v}_0$ and initial acceleration vector $\vec{a}_0$.

(16) Define the state of velocity of a rigid body. Which quantities can the state of velocity of a rigid body be determined unambiguously?

The state of velocity of a rigid body is the set of velocity vectors of the points of the body in a given moment.

Given by:

- angular velocity $\vec{\omega}$
- velocity of (any) point A of the body $\vec{v}_A$

(17) Define the state of acceleration of a rigid body. Which quantities can the state of acceleration of a rigid body be determined unambiguously?

The state of acceleration of a rigid body is the set of acceleration vectors of the points of the body in a given moment.

Given by:

- angular acceleration $\vec{\varepsilon}$
- angular velocity $\vec{\omega}$
- acceleration of (any) point A of the body $\vec{a}_A$

(18) Define translational and rotational motion of a rigid body.

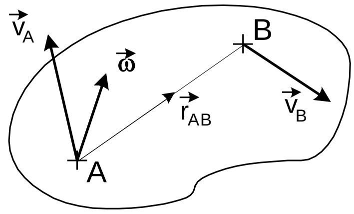
Translational motion or translation: displacement of every point of the body is the same, the body moves parallel to itself.
Rotational motion or rotation: the points of the body move along concentric circles around an axis (axis of rotation) defined by two non-moving points of the body.

(19) Define elementary movement and finite movement of a rigid body.

Elementary movement: movement of the body in an infinitely short time (at a given moment).
Finite movement: movement of the body in longer time.

(20) Give the relation between velocity vectors of two points of a rigid body at a given time.

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{AB}$$



where

- $\vec{\omega}$ - is the angular velocity
- $\vec{v}_A, \vec{v}_B$ - velocities of point A and B
- $\vec{r}_{AB}$ - position vector pointing from point A to B

(21) How can the elementary movements of a rigid body classified?

$\vec{\omega} = 0$	$\vec{v}_A = \vec{0}$	elementary rest
	$\vec{v}_A \neq \vec{0}$	elementary translation
$\vec{\omega} \neq 0$	$\vec{\omega} \cdot \vec{v}_A = 0$	elementary rotation
	$\vec{\omega} \cdot \vec{v}_A \neq 0$	elementary helical motion

(22) Define pole of velocity P and pole of acceleration Q of a rigid body.

The velocity pole is point P of the body that has zero velocity.
The acceleration pole is point Q of the body that has zero acceleration.

(23) Give the definition of the velocity diagram of planar motion of a rigid body. What is the most important theorem concerning to velocity diagram?

The velocity diagram of planar motion is obtained by plotting the velocity vectors of characteristic points of the body from a common starting point in a given moment.

Theorem: The velocity diagram is similar to the position diagram, but it is rotated relative to the position diagram with 90° in the direction of angular velocity.

(24) Give the relation between acceleration vectors of two points of a rigid body at a given time for spatial motion and for planar motion.

Spatial motion:

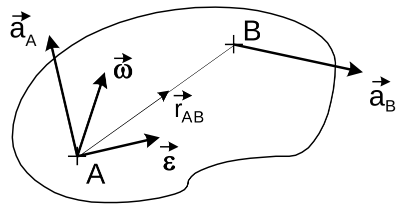
$$\vec{a}_B = \vec{a}_A + \vec{\varepsilon} \times \vec{r}_{AB} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{AB} \right),$$

Planar motion (in plane xy):

$$\vec{a}_B = \vec{a}_A + \varepsilon \vec{k} \times \vec{r}_{AB} - \omega^2 \vec{r}_{AB},$$

Here

- $\vec{\omega}$ - is the angular velocity
- $\vec{\varepsilon}$ - is the angular acceleration
- $\vec{a}_A, \vec{a}_B$ - accelerations of point A and B
- $\vec{r}_{AB}$ - position vector pointing from point A to B



(25) Give the definition of the acceleration diagram of planar motion of a rigid body. What is the most important theorem concerning to acceleration diagram?

The acceleration diagram of planar motion is obtained by plotting the acceleration vectors of characteristic points of the body from a common starting point in a given moment.

Theorem: The acceleration diagram is similar to the position diagram, but it is rotated relative to the position diagram with $180° - \varphi$ in the direction of angular acceleration. φ can be determined by means of the angular velocity and angular acceleration from the relation $\mathrm{tg}\varphi = \varepsilon / \omega^2$.

(26) Define the pure rolling of a rigid body and its consequence.

The instantaneous velocity of the point of contact of the rigid body with the ground is zero in case of pure rolling.

Consequence: the point of contact is instantaneous velocity pole of the rigid body.

(27) When does one talk about relative motion?

If the characteristic quantities of a motion are to be determined in two coordinate systems which move relative to each other.

(28) What are the absolute velocity and acceleration?What are the relative velocity and acceleration?

Absolute velocity and acceleration of a point mass are measured relative to the non-moving coordinate system.

Relative velocity and acceleration of a point mass are measured relative to the moving coordinate system.

(29) What is the relationship between the absolute and relative velocity of a point mass? Specify the meaning and calculation of the quantities in the relation.

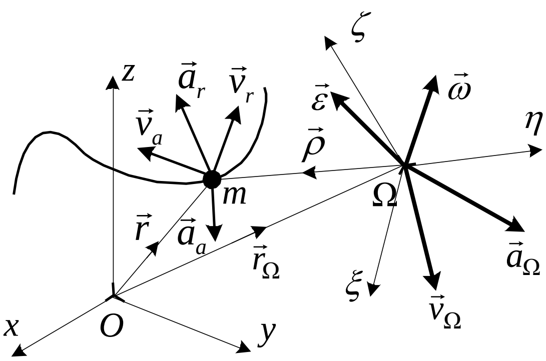
$\vec{v}_a = \vec{v}_r + \vec{v}_t$

$\vec{v}_a$ - velocity of the point mass in the non-moving coordinate system,

$\vec{v}_r$ - velocity of the point mass in the moving coordinate system,

$\vec{v}_t = \vec{v}_\Omega + \vec{\omega} \times \vec{\rho}$.

Velocity of transportation $\vec{v}_t$ is the velocity of the point of the moving coordinate system (measured in the non-moving coordinate system) in which the point mass is at the time being studied.



(30) What is the relationship between the absolute and relative acceleration of a point mass? Specify the meaning and calculation of the quantities in the relation.

$\vec{a}_a = \vec{a}_r + \vec{a}_t + \vec{a}_c$,

$\vec{a}_a$ - acceleration of the point mass in the non-moving coordinate system,

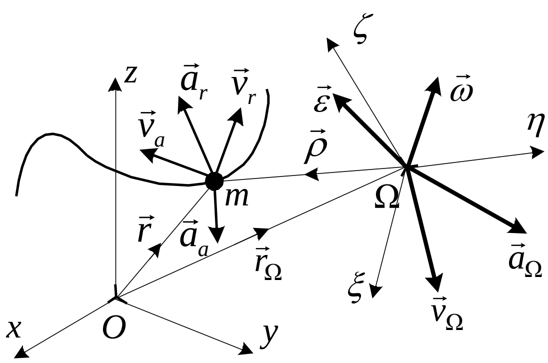
$\vec{a}_r$ - acceleration of the point mass in the moving coordinate system,

$\vec{a}_t = \vec{a}_\Omega + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})$.

Acceleration of transportation $\vec{a}_t$ is the acceleration of the point of the moving coordinate system (measured in the non-moving coordinate system) in which the point mass is at the time being studied.

$\vec{a}_c = 2\vec{\omega} \times \vec{v}_r$.

Coriolis accelerationoccursif $\vec{\omega} \neq \vec{0}$, $\vec{v}_r \neq \vec{0}$ and $\vec{\omega}$ is not $\parallel \vec{v}_r$.



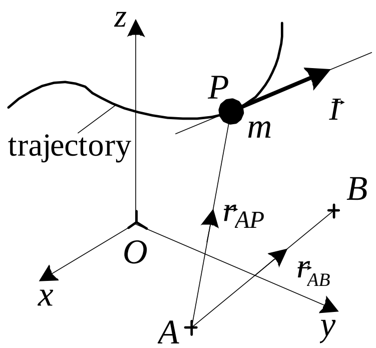
(31) Define the linear and angular momentum of a point mass.

The linear momentum of a point mass is equal to the product of the mass and the velocity of the point mass:

$$\vec{I} = m\vec{v} \, .$$

The angular momentum of a point mass calculated for standing point A is equal to the moment of the linear momentum of the point mass calculated for point A:

$$\vec{\pi}_A = \vec{r}_{AP} \times \vec{I} \, .$$



(32) Define the kinetic energy of a point mass. Define the power of a force acting on a point mass.

The kinetic energy of a point mass is half of the product of the mass and the square of the velocity of the point mass:

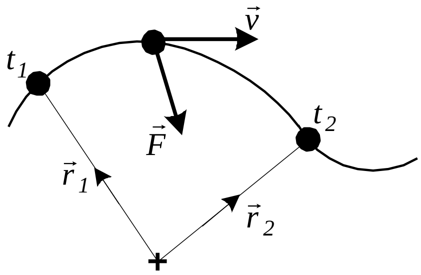
$$E = \frac{1}{2} m v^2 \, .$$

The power of a force acting on a point mass is the dot product of the force and velocity of the point mass:

$$P = \vec{F} \cdot \vec{v} \, .$$

The work of a force acting on a point mass in time interval <t_1 ,t_2> is the integral of the power of the force from t_1 tot_2:

$$W_{12} = \int_{t_1}^{t_2} P \, dt = \int_{t_1}^{t_2} \vec{F} \cdot \underbrace{\vec{v} \, dt}_{d\vec{r}} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} \ \ .$$



(33) What is Newton's first law of motion?

In an inertial frame of reference, an object either remains at rest or continues to move at a constant velocity, unless acted upon by a force.

(34) What is Newton's second law of motion generally and specially in engineering practice?

Generally: the derivative of linear momentum of a point mass with respect to time is equal to the resultant force of the forces acting on the point mass: $\dot{\vec{I}} = \frac{d}{dt}\left(m\vec{v}\right) = \vec{F}$.

In engineering practice$m$ = constant usually: $m\vec{a} = \vec{F}$.

(35) What is Newton's third law of motion?

The magnitude of the effect of two bodies on each other is always equal and the direction of the effects is always opposite: $\vec{F}_{12} = -\vec{F}_{21}$.

(36) What is D'Alembert's principle?

Kinetic problems can be traced back to static problems by introducing inertial forces:

$$\vec{0} = -m\vec{a} + \vec{F} = \vec{T} + \vec{F} \ \ ,$$

$\vec{T}$ - inertial force

$\vec{F}$ - resultant force of the external forces.

(37) Write down the differential and integral forms of the angular momentum theorem of a point mass.

Differential form: $\dot{\vec{\pi}}_A = \vec{M}_A$.

The first derivative of angular momentum of a point mass with respect to time equals to the resultant moment of the forces acting on the point mass.

Integral form: $\vec{\pi}_A(t_2) - \vec{\pi}_A(t_1) = \int_{t_1}^{t_2} \vec{M}_A dt$.

The change of the angular momentum of a point mass in the time interval <t_1,t_2>equals to the integral of moment of the forces acting on the point mass from t_1 to t_2.

Reference point *A*is at rest and it is the same point for the angular momentum and for the moment.

(38) Write down the theorem of mechanical energy and the theorem of work for a point mass.

The theorem of mechanical energy: $\dot{E} = P$.
The derivative of kinetic energy of a point mass with respect to time equals to the power of forces acting on the point mass.

The theorem of work: $E_2 - E_1 = W_{12}$.
The change of kinetic energy of a point mass in the time interval <t_1,t_2>equals to the work of forces acting on the point in the same time interval.

(39) Give the definition of conservative force field. What does work depend in conservative filed on?

A force system (force field) is called conservative if there is a scalar function (named potential field) $U = U(\vec{r})$ from which the force field can be derived as follows: $\vec{F} = -\frac{dU}{d\vec{r}} = -gradU$.

Work in the conservative force field depends only on the initial and final positions. It is equal to the difference between the values of the potential$U = U(\vec{r})$ in the initial and final point: $W_{12} = U(\vec{r}_1) - U(\vec{r}_2)$.

(40) What is the theorem of conservation of mechanical energy?

The sum of kinetic and potential energy is constant during motion in conservative field:$E+U$ = constant.

(41) Define the free and constrained motion.

Free motion: if the motion of the body studied is not obstructed by other bodies.

Constrained motion: if the motion of the body studied is restricted by other bodies in according to the prescribed geometrical conditions.

(42) What kind of constraint force occurs in case of smooth and rough constraints?

In case of smooth constraints the constraint force is perpendicular to the contact surfaces.

In case of rough constraints*Coulomb's law of friction*creates connection between the normal and tangential coordinates of the constraint force.

(43) What is kinetic friction? Write down Coulomb's law of friction for kinetic friction.

Kinetic friction: if there is a relative tangential displacement between the contactpoints.

Coulomb's law of friction: the tangential coordinate of the constraint force is equals to μ times the normal coordinate and its direction is opposite to the direction of the velocity. (μ is the coefficient of the kinetic friction.)

Tangential component: $\vec{F}_S = -\mu F_N \dfrac{\vec{v}}{|\vec{v}|}$.

Tangentialcoordinate: $F_S = -\mu F_N$.

(44) Define inertial and non-inertial frame.

Inertial frame is a coordinate system in which motion can be explained by the interaction of other bodies, that is by considering only external forces.

Non-inertial frame is a coordinate system in which further forces are required to describe motion in addition to external forces.

(45) Write down fundamental law of kinetics for a point mass in a non-inertial system and give the meaning of the quantities in it.

$\vec{F}_a + \vec{F}_t + \vec{F}_c = \vec{F}_r$.

Force in the stationary (standing) coordinate system:

$\vec{F}_a = m\vec{a}_a$.

Force in the moving coordinate system:

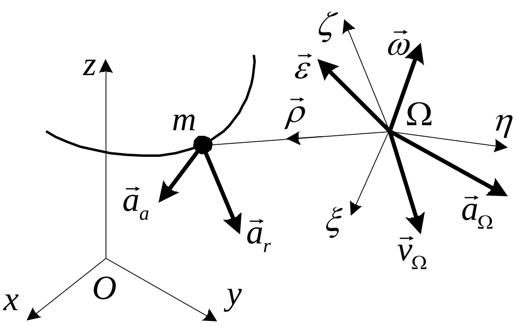
$\vec{F}_r = m\vec{a}_r$.

Force of transportation:

$\vec{F}_t = -m\vec{a}_t = -m\left[\vec{a}_\Omega + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times \left(\vec{\omega} \times \vec{\rho}\right)\right]$.

Coriolis force:

$\vec{F}_c = -m\vec{a}_c = -2m\left(\vec{\omega} \times \vec{v}_r\right)$.



(46) What are the definitions of a continuum, a body with homogeneous mass distribution, and a body with discrete mass distribution?

Continuum: a body whose material fills its volume continually.

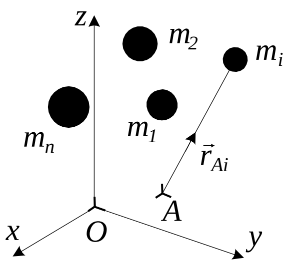
Body with homogeneous mass distribution: a body (a continuum) whose mass density is constant (independent of the location).

Body with discrete mass distribution: a body consists of points of material which are fixed to a rigid frame with negligible mass.

(47) Define static moment of a body with discrete mass distribution and a continuum about a reference point. What is the relation between static moments about different points?

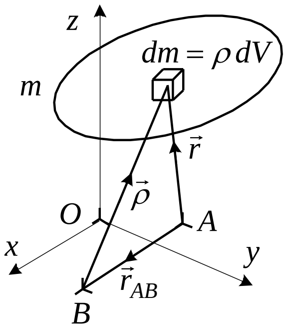
Static moment of a body with discrete mass distributionabout point *A*:

$$\vec{S}_A = \sum_{i=1}^{n} \vec{r}_{Ai} m_i \, .$$



Static moment of a continuumabout point *A*:

$$\vec{S}_A = \int_{(m)} \vec{r} \, dm = \int_{(V)} \vec{r} \rho \, dV \, .$$



Relation between static moments about different points:

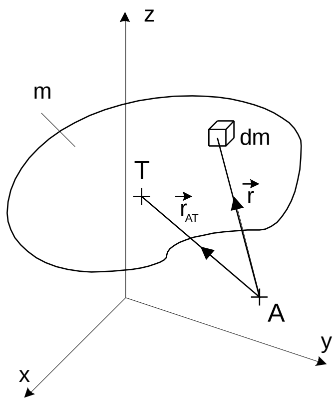
$$\vec{S}_B = \vec{S}_A - m \vec{r}_{AB} \, .$$

(48) Define center of mass of a body. How is the position vector of center of mass calculated?

Center of mass of a body is point *T* about which the static moment is zero: $\vec{S}_T = \vec{0}$.

Position vector of center of mass:

$$\vec{r}_{AT} = \frac{\vec{S}_A}{m} = \frac{\int_{(V)} \vec{r} \rho \, dV}{m} = \frac{\sum_i m_i \vec{r}_{Ai}}{m} \, .$$



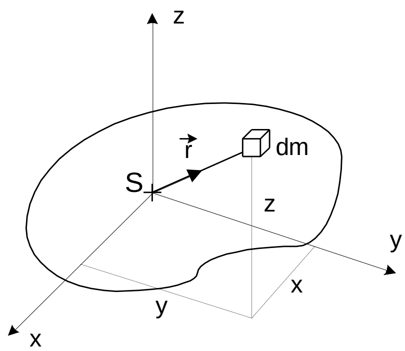
(49) Give an interpretation of the principal moments of inertia around axes x, y, z with respect to point S of a rigid body. Define the products of inertia.

Principal moments of inertia: Products of inertia:

$$J_x = \int_{(m)} \left(y^2 + z^2\right) dm \, , \qquad J_{xy} = \int_{(m)} xy \, dm \, ,$$

$$J_y = \int_{(m)} \left(x^2 + z^2\right) dm \, , \qquad J_{xz} = \int_{(m)} xz \, dm \, ,$$

$$J_z = \int_{(m)} \left(x^2 + y^2\right) dm \, , \qquad J_{yz} = \int_{(m)} yz \, dm \, .$$



(50) *Write down the tensor of moment of inertia about center of mass of a body with continuous mass distribution. What is its most important feature?*

Tensor of moment of inertia about center of mass S: $\left[\underline{\underline{J}}_S\right] = \begin{bmatrix} J_x & -J_{xy} & -J_{xz} \\ -J_{yx} & J_y & -J_{yz} \\ -J_{zx} & -J_{zy} & J_z \end{bmatrix}$.

It is symmetric tensor.

(51) *How can the inertia J_n and J_{nm} be calculated knowing the tensor of moment of inertia $\underline{\underline{J}}_S$ about the center of mass and the unit vectors $\vec{n}$ and $\vec{m}$?*

$$J_n = \vec{n} \cdot \underline{\underline{J}}_S \cdot \vec{n} , \qquad J_{nm} = J_{mn} = -\vec{n} \cdot \underline{\underline{J}}_S \cdot \vec{m} = -\vec{m} \cdot \underline{\underline{J}}_S \cdot \vec{n} .$$

(52) *What are the principal axes and principal moments of inertia of a body?*

If the following conditions are met

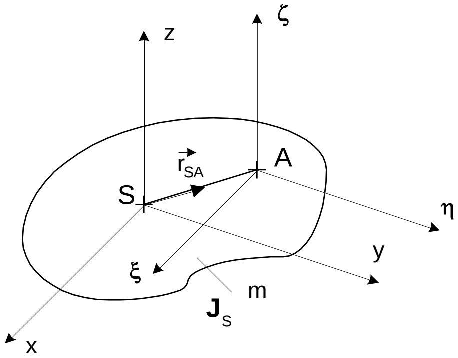
$$\underline{\underline{J}}_S \cdot \vec{e}_1 = J_1\vec{e}_1 , \quad \underline{\underline{J}}_S \cdot \vec{e}_2 = J_2\vec{e}_2 , \quad \underline{\underline{J}}_S \cdot \vec{e}_3 = J_3\vec{e}_3 ,$$

then unit vectors $\vec{e}_1, \ \vec{e}_2, \ \vec{e}_3$ are the directions of the principal axes and scalars $J_1, \ J_2, \ J_3$ are the principal moments of inertia.

(53) *Write down Steiner's theorem and draw an explanatory figure.*

Steiner's theorem:

$$J_\xi = J_x + m\left(y_{SA}^2 + z_{SA}^2\right), \quad J_{\xi\eta} = J_{xy} + m x_{SA} y_{SA} ,$$
$$J_\eta = J_y + m\left(x_{SA}^2 + z_{SA}^2\right), \quad J_{\xi\zeta} = J_{xz} + m x_{SA} z_{SA} ,$$
$$J_\zeta = J_z + m\left(y_{SA}^2 + x_{SA}^2\right), \quad J_{\eta\zeta} = J_{yz} + m z_{SA} y_{SA} .$$



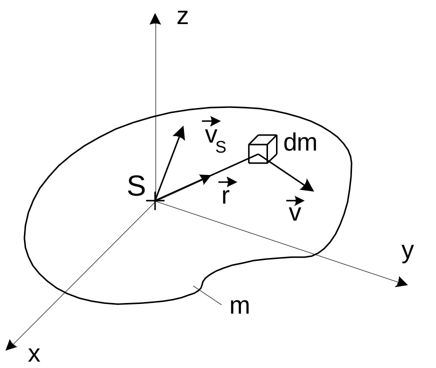
(54) *Describe the relation between the principal axes of moment of inertia and symmetry properties for a body with homogeneous mass distribution.*

- If the body has a plane of symmetry, then axis perpendicular to the plane of symmetry and passing through the center of mass is principal axis of moment of inertia.
- If the body has two planes of symmetry, then the line of intersection of the planes is principal axis of moment of inertia.
- The intersection lines of three perpendicular planes of symmetry are principal axes of moment of inertia.
- In the case of axis symmetry the axis of symmetry and all axes in plane perpendicular thereto are principal axes of moment of inertia.

(55) Define linear momentum of a rigid body. Write down the relation used to calculate it if the reference point is the center of mass.

Definition: $\vec{I} = \int\limits_{(m)} \vec{v}\, dm$.

Calculation: $\vec{I} = m\vec{v}_S$.

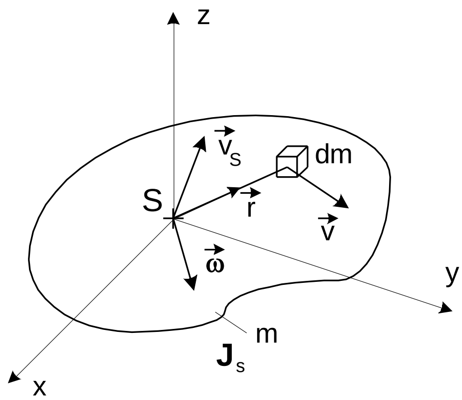


(56) Define angular momentum about center of mass of a rigid body. Write down the relation used to calculate it.

Definition: $\vec{\pi}_S = \int\limits_{(m)} \vec{r} \times \vec{v}\, dm$.

Calculation:

$$\vec{\pi}_S = \underline{\underline{J}}_S \cdot \vec{\omega} \text{ where } \underline{\underline{J}}_S = \begin{bmatrix} J_x & -J_{xy} & -J_{xz} \\ -J_{yx} & J_y & -J_{yz} \\ -J_{zx} & -J_{zy} & J_z \end{bmatrix}.$$

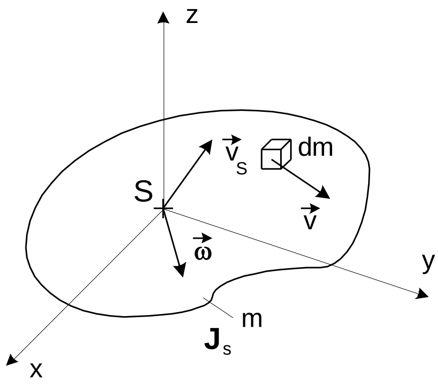


(57) Define kinetic energy of a rigid body. Write down the relation used to calculate it if the reference point is the center of mass.

Definition: $E = \frac{1}{2} \int\limits_{(m)} v^2 dm$.

Calculation:

$$E = \frac{1}{2}\left(\vec{v}_S \cdot \vec{I} + \vec{\omega} \cdot \vec{\pi}_S\right) = \frac{1}{2} m v_S^2 + \frac{1}{2} \vec{\omega} \cdot \underline{\underline{J}}_S \cdot \vec{\omega} .$$



(58) How is the power of force system acting on a rigid body calculated by means of the resultant force and resultant moment? How is the power of the same force system acting calculated by means of the forces and moments?

- by means of the resultant force and resultant moment: $P = \vec{F} \cdot \vec{v}_S + \vec{M}_S \cdot \vec{\omega}$.

$\vec{F}$, $\vec{M}_S$ - resultant force and resultant moment about point *S*,

$\vec{\omega}$, $\vec{v}_S$ - angular velocity and velocity of point *S*of the body.

- by means of the forces and moments: $P = \sum_{i=1}^{n} \vec{F}_i \cdot \vec{v}_i + \sum_{j=1}^{m} \vec{M}_j \cdot \vec{\omega}_j$.

n -number of the concentrated forces of the force system,

m - number of the concentrated moments of the force system,

$\vec{v}_i$ - velocity of the point of application of concentrated force $\vec{F}_i$,

$\vec{\omega}_j$ - angular velocity of rigid body on which concentrated moment $\vec{M}_j$ acts.

(59) Write down the differential form of the theorem of linear momentum and the theorem of angular momentum about center of mass for a rigid body. What are the quantities in the expressions?

Differential form of the theorem of linear momentum: $\dot{\vec{I}} = \vec{F}$ or $m\vec{a}_S = \vec{F}$ where

$\dot{\vec{I}}$ - time derivative of linear momentum of the body

$\vec{F}$ - resultant force of external forces acting on the body

m - mass of the body

$\vec{a}_S$ - acceleration of point *S*of the body

Differential form of the theorem of angular momentum about point *S*:

$$\dot{\vec{\pi}}_S = \vec{M}_S \text{ or } \underline{\underline{J}}_S \cdot \vec{\varepsilon} + \vec{\omega} \times \vec{\pi}_S = \vec{M}_S \text{ where}$$

$\dot{\vec{\pi}}_S$ - time derivative of angular momentum about point *S*

$\vec{M}_S$ - resultant moment of external forces acting on the body about point *S*

$\underline{\underline{J}}_S$ - moment of inertia of the body about point *S*

$\vec{\varepsilon}$ - angular acceleration of the body

$\vec{\omega}$ - angular velocit of the body

(60) Write down the integral form of the theorem of linear momentum and the theorem of angular momentum about center of mass for a rigid body. What are the quantities in the expressions?

Integral form of the theorem of linear momentum: $\Delta\vec{I} = \vec{I}(t_2) - \vec{I}(t_1) = \int_{t_1}^{t_2} \vec{F}(t)dt$ where

$\vec{I}(t_1)$ - linear momentum of the body at moment t_1

$\vec{F}(t)$ - resultant force of external forces acting on the body

Integral form of the theorem of angular momentum about point *S*:

$$\Delta\vec{\pi}_S = \vec{\pi}_S(t_2) - \vec{\pi}_S(t_1) = \int_{t_1}^{t_2} \vec{M}_S(t)\,dt \text{ where}$$

$\vec{\pi}_S(t_1)$ - angular momentum of the body about point *S*at moment t_1

$\vec{M}_S(t)$ - resultant moment of external forces acting on the body about point *S*

(61) Write down the differential form of the theorem of angular momentum about (any) point Afor a rigid body. What arethe quantities in the expression?

$$\underline{\underline{J}}_A \cdot \vec{\varepsilon} + \vec{\omega} \times \left(\underline{\underline{J}}_A \cdot \vec{\omega}\right) + \vec{r}_{AS} \times m\vec{a}_A = \vec{M}_A \text{ where}$$

$\underline{\underline{J}}_A$ - moment of inertia of the body about point *A*

$\vec{\varepsilon}$ - angular acceleration of the body

$\vec{\omega}$ - angular velocity of the body
m - mass of the body
$\vec{r}_{AS}$ - position vector points from point A to the center of mass
$\vec{v}_A$ - velocity of point A of the body
$\vec{a}_A$ - acceleration of point A of the body
$\vec{M}_A$ - resultant moment of external forces acting on the body about point A

(62) Write down the theorem of mechanical energy and theorem of work for a rigid body.

Theorem of mechanical energy: time derivative of kinetic energy of a rigid body equals to the power of external force system acting on the body:

$$\dot{E} = P .$$

Theorem of work: change in kinetic energy of a rigid body during finite motion of the body is equal to the work of an external force system acting on the body during the same motion:

$$E_2 - E_1 = W_{12} .$$

(63) When is a body rotating about a fixed axis said to be statically and dynamically balanced?

A rigid body rotating about a fixed axis is said to be statically balanced if the center of mass of the body lies on the axis of rotation.

A rigid body rotating about a fixed axis is said to be dynamically balanced if the axis of rotation is principal axis of moment of inertia of the body.

(64) Under what assumptions have the impacts/collisions of bodies been studied?

- the colliding bodies are elastic in some extent
- the collisions take a very short time
- there is no change in the position of the bodies during brief contact
- forces not related to the collisions are negligible
- contact surfaces are smooth(no friction)

(65) What are the central impact, direct central impact, oblique central impact, and eccentric impact?

Central impact: the centres of mass of both bodies lie on the line of impact.

Direct central impact: the velocities of the centres of massof the bodies point in the direction of the line of impact.

Oblique central impact: the velocities of the centres of mass of the bodies do not point in the direction of the line of impact.

Eccentric impact: at least one of the centres of mass of the bodies does not lie on the line of impact.

(66) Define coefficient of restitution for central impact.

Coefficient of restitution is the ratio between the change of linear momentum in the restoration phase and the change of linear momentum in the deformationphase:

$$k = \frac{m_1\left(V_{S1n} - v_{Sn}\right)}{m_1\left(v_{Sn} - v_{S1n}\right)} = \frac{m_2\left(V_{S2n} - v_{Sn}\right)}{m_2\left(v_{Sn} - v_{S2n}\right)} .$$

(67) What are the simple and complex structures?

Simple structure: system (structure) studied consists of only one rigid body.

Complex structure: system (structure) studied consists of several rigid body.

(68) Define the notion of generalized coordinate q(t). What are its most important features?

Generalized coordinate: scalar parameters (coordinates) *q(t)* that determine the position or motion of a system unambiguously.

Features:

- it can be displacement or angle
- it is a function of time whichis at least twice differentiable($\ddot{q}$ exists)

(69) Specify the properties of an ideal rope.

- weightless and unstretchable
- perfectly flexible (no bending resistance - no bending moment)
- only axial tensile can occur in it (no compression)