SZÉCHENYI ISTVÁN UNIVERSITY

DEPARTMENT OF APPLIED MECHANICS

MECHANICS – STATICS

Theoretical questions and answers for BSc students

1) *What does the permanent rest mean?*

The position of a body (studied body) is not changing for long time.

2) *What does modelling mean?*

A kind of idealisation (simplification) which keeps the important features of a studied system, phenomenon and ignores the unimportant ones. (It depends on the given studies what important is.)

3) *What kind of simplification of a material model? What does it assume?*

It assumes a simpler behaviour of the material than it has in reality.

4) *Define the concept of a rigid body.*

It is body model in which the distance between any two points is constant. (The distance between points does not change under load.)

5) *Define the concept of a solid body.*

There can be deformations in this body model. (The distance between points can change under load.)

6) *What kind of simplification of a geometrical model? What does it assume?*

A geometrical model assumes that the form of the studied body is simpler than in reality.

7) *Define the concept of a point mass.*

1st definition: geometrical point with material features.

2nd definition: a body whose position (motion) can be defined by the position (motion) of one of its points unambiguously.

8) *Define the concept of a rod.*

It is a body whose one dimension is much larger than the other two.

9) *What is a force?*

It is an effect of a body on another body (shifting).

10) *What is a force system?*

A set of forces which are related to each other in some sense. (For example they affect the same body.)

11) *What is a load?*

The effect of bodies not belonging to the system.

12) *What are the concentrated force and distributed force over a line?*

Concentrated force is an effect from the point of contact of two bodies.

Distributed force over a line is a force over a contact line between two bodies.

13) *What are the traction (distributed force on a surface) and body force (distributed force in a volume)?*

Traction is a force over a contact surface between two bodies.

Body force is an interaction between bodies which is intermediated by a field.

14) *What are the constraints and the support forces?*

Constraints are connections between the studied body and its surroundings (or between bodies of the system) which obstruct the movement of the body.

Support forces are unknown effects of the constraints that affect the system.

15) *What kind of movement is prevented by a hinge(d or pinned support)?*

It prevents any movement of a point of the studied body/rod.

16) *What kind of movement is prevented by a roller (movable support)?*

It prevents the movement of a point of the studied body/rod in a given direction.

17) *What kind of movement is prevented by a support with rod/suspension with rope?*

It prevents the movement of a point of the studied body/rod in a given direction.

18) *What kind of movement is prevented by fixed a support?*

It prevents any movement of any point of the studied body/rod in any direction and it prevents any rotation of any cross section of the studied body/rod around every axis.

19) *Define the concept of degree of freedom. Summarize the degree of freedom of a point mass and a rigid body in a table. Include 2D and 3D case, too.*

Degree of freedom is the number of scalar data (coordinates) that defines the position of a body unambiguously.

	2D	3D
point mass	2	3
rigid body	3	6

20) *When are two force systems (with a common point of application / acting on the same point mass) equivalent? What is an equilibrium force system with a common point of application?*

-Two force systems with a common point of application are equivalent to each other if the resultant forces are equal.

$\vec{F}' = \vec{F}''$ where $\vec{F}' = \sum_{i=1}^{n} \vec{F}_i'$ and $\vec{F}'' = \sum_{j=1}^{m} \vec{F}_j''$ are the resultant vectors of the force systems.

-An equilibrium force system with a common point of application has zero resultant vector: $\vec{F} = \sum_{i=1}^{n} \vec{F}_i = \vec{0}$

21) *What is the fundamental theorem of statics on a mass point?*

A point mass remains at rest if the force system acting on it is in equilibrium:

$$\vec{F} = \sum_{i=1}^{n} \vec{F}_i = \vec{0}.$$

22) *What are the criteria of the equilibrium of two forces?*

Two forces are in equilibrium if
- their line of action is the same,
- their magnitudes (absolute values) are the same,
- their directions are opposite.

23) *What are the criteria of the equilibrium of three forces?*

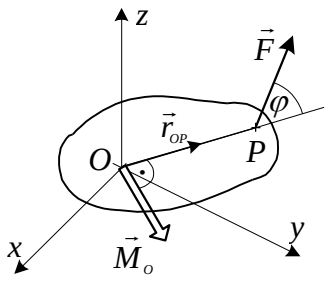
Three forces can be in equilibrium only if

- their lines of action are in the same plane and intersect in a common point,

- the force vectors are on the sides of a closed vector triangle,

- the arrows of the forces of the closed triangle return to the starting point.

24) *What is the definition of moment/torque of a force about a point?*



Moment of a force about a point: rotating effect of a force about a given point.

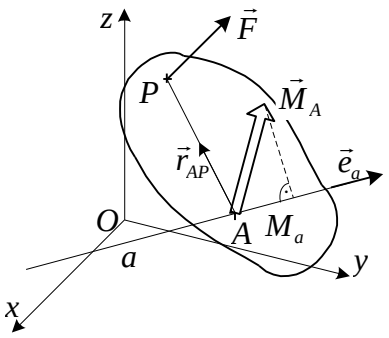
$\vec{M}_O = \vec{r}_{OP} \times \vec{F}$.

$\vec{M}_O$ – moment vector about point *O*,

$\vec{r}_{OP}$ – position vector points from point *O* to point *P* (where *P* is the point of application of the force),

$\vec{F}$ – force vector.

25) *What is the definition of moment/torque of a force about an axis?*



Moment of a force about an axis:

rotating effect of a force about a given axis.

$M_a = \vec{M}_A \cdot \vec{e}_a$.

A – an arbitrary point of axis *a*,

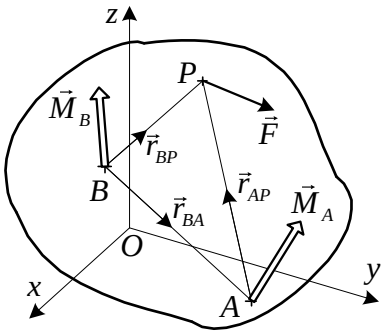
$\vec{e}_a$ – direction unit vector of axis *a*.

26) *On which geometric objects is the moment zero?*

- about points lying on the line of action of the force (because $\vec{r}_{AP} \parallel \vec{F}$),

- about axes intersecting the line of action of the force (because the moment about the intersecting point is zero),
- about axes parallel to the line of action of the force (because the moment about the points of the axis is perpendicular to the axis).

27) *What is the relation between moments about two different points?*



$\vec{r}_{BP} = \vec{r}_{BA} + \vec{r}_{AP} = \vec{r}_{AP} + \vec{r}_{BA}$

From the definition of the moment:

$\vec{M}_B = \vec{r}_{BP} \times \vec{F} = \underbrace{\vec{r}_{AP} \times \vec{F}}_{\vec{M}_A} + \vec{r}_{BA} \times \vec{F}$.

$\vec{M}_B = \vec{M}_A + \vec{r}_{BA} \times \vec{F}$, or $\vec{M}_B = \vec{M}_A + \vec{F} \times \vec{r}_{AB}$.

28) *What is the definition of an equivalent force couple system of a force system?*

It has the same effect as the force system.

$\vec{F} = \sum_{i=1}^{n} \vec{F}_i$, $\vec{M}_A = \sum_{j=1}^{m} \vec{M}_j + \sum_{i=1}^{n} \vec{r}_{Ai} \times \vec{F}_i$

where $\vec{F}_1, \vec{F}_2, \vec{F}_3,$ … , $\vec{F}_i,$ … , $\vec{F}_n$ are the concentrated forces of the force system and $\vec{M}_1, \vec{M}_2, \vec{M}_3,$ … , $\vec{M}_j,$ … , $\vec{M}_m$ are the concentrated moments of the force system.

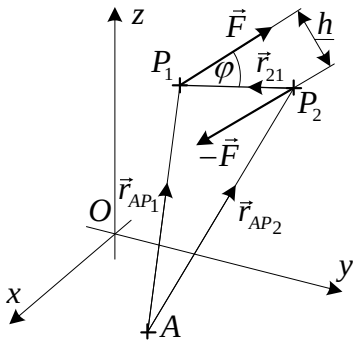
29) *When are two force systems (non-concurrent / acting on the same rigid body) equivalent? What is an equilibrium force system acting on a rigid body?*

-Two force systems acting on a rigid body are equivalent if their moment about any point are the same, that is their resultants are the same $\vec{F}' = \vec{F}''$ and $\vec{M}'_A = \vec{M}''_A$.

- Criteria of the equilibrium:

$\vec{F} = \sum_{i=1}^{n} \vec{F}_i = \vec{0}$, and $\vec{M}_A = \sum_{i=1}^{n} \vec{r}_{Ai} \times \vec{F}_i = \vec{0}$.

30) *Give the definition of a couple (concentrated moment).*



It is a force system of two parallel forces of equal magnitude acting in opposite direction.

$\vec{M}_A = \vec{r}_{21} \times \vec{F}$

31) *What is the fundamental theorem of statics (of a rigid body)?*

A rigid body remains at rest if the force system acting on it is in equilibrium.
Criteria of the equilibrium:

$\vec{F} = \sum_{i=1}^{n} \vec{F}_i = \vec{0}$, and $\vec{M}_A = \sum_{i=1}^{n} \vec{r}_{Ai} \times \vec{F}_i = \vec{0}$. Point *A* can be any point of the space.

32) *What are the properties of a couple?*

Properties of a couple (concentrated moment):
- magnitude of the moment of a couple about any point of the space is the same,
- magnitude of the moment equals to the product of the magnitude of the force and the perpendicular distance of the lines of action,
- direction of the moment is perpendicular to the plane of the lines of action,
- direction of the moment can be obtained by the right-hand rule.

33) *What is the generalization of the concept of a force system?*

It is a set of related (e.g. acting on the same body) forces and moments of couples.

34) *Define a coplanar force system.*
All the vectors of a coplanar force system lie in the same plane.

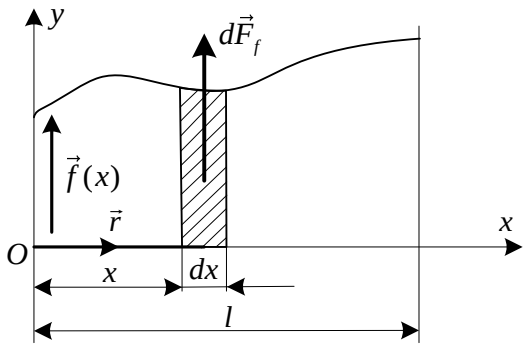
35) *What is a non-concurrent coplanar force system?*

If a coplanar force system consists of more than two forces and the lines of action of the forces do not intersect in a common point, then the force system is called non-concurrent coplanar force system.

36) *What is equivalent with a non-concurrent coplanar force system? What is the points of the line of action of the resultant force characterized?*

A non-concurrent coplanar force system is equivalent to one force (the resultant force) or to one couple (if the resultant force is zero). If the resultant force is not zero, then the moment of the force system about any point of the line of action of the resultant force of the force system is zero.

37) *There is a force system distributed along a line. How can its resultant force and resultant moments (about a point O) be calculated?*



Density vector of a distributed force system over a line is:

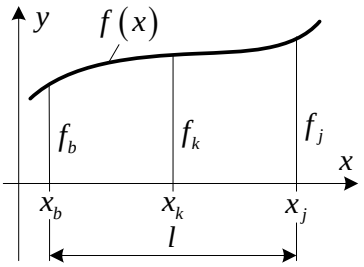
$\vec{f}(x) = f_y(x)\,\vec{j}$.

The resultant force: $\vec{F} = \int_{(l)} \vec{f}(x)\,dx$.

The resultant moment about point *O* is:

$\vec{M}_O = \int_{(l)} \vec{r} \times \vec{f}\,dx = \int_{(l)} x\, f_y(x)\,dx\,\vec{k}$

38) *What is Simpson's rule? When does the Simpson's rule yield the exact value of an integral?*



$$\int_{(l)} f(x)\,dx \cong \frac{l}{6}\left(f_b + 4f_k + f_j\right)$$

Simpson's rule yields the exact value of an integral if the integrand is a polynomial of degree three or less.

39) *What is a parallel force system?*

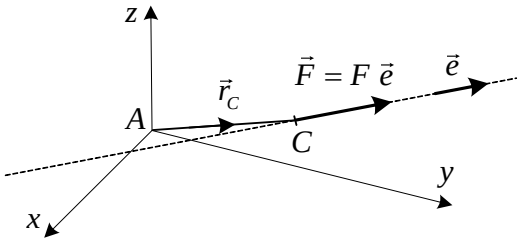
If all the lines of action of forces of a force system are parallel to a given direction $\vec{e}$, it is called parallel force system.

40) *What does the position vector of the point of application of the resultant force* $\left(\vec{F} \neq \vec{0}\right)$ *of a parallel force system depend on?*

If the resultant force of the parallel force system is not zero, its point of application (that is the center of the forces) is independent from the direction of the forces. It depends only on the magnitudes of the forces and position vectors of points of application of the forces.

41) *What is the definition of the center C of forces of a parallel force system? Give its position vector.*

Center of forces *C* of a parallel force system is a point in which the system is replaceable only with one force vector, i.e. its effect is equivalent to the effect of the whole system.



Position vector of the center of forces:

$$\vec{r}_C = \frac{\sum_{i=1}^{n} \vec{r}_i F_i}{\sum_{i=1}^{n} F_i}$$

where *n* is the number of the forces and $\vec{r}_i$ are the position vectors pointing from *O* to the point of application of the froces.

42) *What is the definition of the center of gravity CG? Give its position vector.*

Center of gravity *CG* of a body is the center of distributed weight forces of the body.

Position vector of the center of gravity *GC* of the body: $\vec{r}_C = \vec{r}_{CG} = \frac{\int_{(V)} \vec{r}\,\rho g dV}{\int_{(V)} \rho g dV}$.

Where *g* is the gravitational acceleration and ρ is the mass density.

43) *What is the definition of a rod?What is the mechanical model of a rod?*

A body whose one dimension is much greater than the other two. Centerline is a mechanical model of the rod.

44) *Define the concept of a prismatic rod.*

Definition 1: If the size and shape of the cross sections along the length of the rod do not change.

Definition 2: If the cross sections of the rod can be shifted parallelly into each other along the centerline.

45) *Give the definition of cross section of a rod. Define the centerline of a rod.*

Cross section is a section which is perpendicular to the largest dimension of the rod. Centerline is formed from the the centers of gravity of the cross sections of the rod.

46) *Define the internal forces of a rod.*

There is a distributed load in any cross section of the rod. Internal forces are the equivalent force couple system (or its coordinates) of this load about the center of gravity of the cross section.

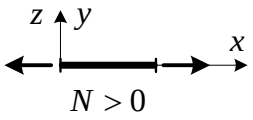
47) *Describe how to determine the internal forces of a cross section.*

- the rod is divided into two sections in the given cross section imaginarily,

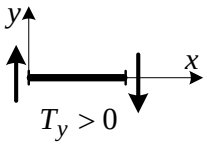
- one part of the rod is left, the other is kept,

- force system acting on the left part is reduced into the center of gravity of the kept part.

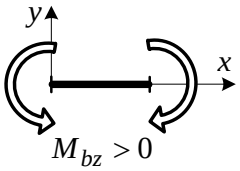
48) *Give interpretation of positive sign of internal force of a planar structure by means of figures.*



positive normal force



positive shear force



positive bending moment

49) *How can diagrams of internal forces be get?What are the internal forces in planar case?*

The internal forces are determined at every cross section of the rod and depicted as a function of position along the rod.

Internal forces in planar case: $N = N(x)$, $T = T(x)$, $M_{bz} = M_{bz}(x)$.

50) *Give the relation between $q_y(x)$ and $T_y(x)$ where $q_y(x)$ is the distributed load along the centerline of the rod and $T_y(x)$ is the shear force. Give the differenetial and integral form of the relation.*

Differential form: $\frac{dT_y(x)}{dx} = q_y(x)$.

Integral form: $T_y(x_2) - T_y(x_1) = \int_{x_1}^{x_2} q_y(x)dx.$

51) *Give the relation between $T_y(x)$ and $M_{bz}(x)$ where $T_y(x)$ is the shear force along the centerline of the rod and $M_{bz}(x)$ is the bending moment. Give the differenetial and integral form of the relation.*

Differential form: $\frac{dM_{bz}(x)}{dx} = -T_y(x).$

Integral form: $M_{bz}(x_2) - M_{bz}(x_1) = -\int_{x_1}^{x_2} T_y(x)dx.$

52) *Write the number of constrained degrees of freedom for a roller, a hinge and a fixed support. Give these numbers for planar and spatial cases, too.*

	planar case	spatial case
roller	1	1
hinge/pin	2	3
fixed support	3	6

53) *What is the definition of a statically determinate structure?*

Definition 1: If the number n_c of the constrained degrees of freedom equals to the total number f of degrees of freedom of the bodies: $n_c=f$. (Constraints can be inner and outer: $n_c=n_i+n_o$.)

Definition 2: If the number of coordinates of the unknown support and inner forces equals to the number of the scalar equations which describe the equilibrium of the structure and the system of equations can be solved unambiguously.

54) *What is the definition of a statically undeterminate structure?*

Definition 1: If the number n_c of the constrained degrees of freedom is greater than the total number f of degrees of freedom of the bodies: $n_c>f$. (Constraints can be inner and outer: $n_c=n_i+n_o$.)

Definition 2: If the number of coordinates of the unknown support and inner forces is greater than the number of the scalar equations which describe the equilibrium of the structure.

55) *Write the number of scalar equilibrium equations for a point mass, a rigid body and a system containing N rigid bodies. Give these numbers for planar and spatial cases, too.*

	planar case	spatial case
point mass	2	3
rigid body	3	6
system containing N rigid bodies	$3N$	$6N$

56) *What is a rod structures with pins?*

The rods of the structure are connected by pins (hinges) and they are connected to the environment by arbitrary constraints. (The pins are not necessarily located at the end of the rods.)

57) *What is a planar truss?*

Beam truss: it is made only straight beams (rods) which are connected with pins (hinges). Loads and support forces act only on the pins. (Individual beams/rods are called members, too.)

Planar truss: all the centerlines of the rods lie in a plane.

58) *Describe the method of joints with which the member forces can be determined.*

- Equilibrium of a given node is studied.

- The unknown member forces are determined by the equilibrium equations of the coordinates of the resultant force.

Two scalar unknowns can be determined by this method in planar case.

59) *Describe the method of sections with which the member forces can be determined.*

- The structure is divided into two sections imaginarily.

- Keep one section and leave the other.

- There occur member forces in the cut rods of the kept.

- The member forces are determined from the equilibrium of the kept section.

- Three scalar unknowns can be determined by this method in planar case.

60) *What is static friction?Give the relation between the normal and the tangential component of the support force.*

If the contact surfaces are rough and there is not relative tangential displacement between the contact points.

$F_t \le \mu_0 F_n$ where F_n is the normal coordinate of the support force, F_t is the tangential coordinate of the support force, and μ_0 is the coefficient of static friction.

61) *What is kinetic friction?Give the relation between the normal and the tangential component of the support force.*

If the contact surfaces are rough and there is a relative tangential displacement between the contact points.

$F_t = \mu F_n$ where F_n is the normal coordinate of the support force, F_t is the tangential coordinate of the support force, and μ is the coefficient of kinetic friction.

62) *What is the definition of pure rolling?What are its criteria?*

It is rolling without relative tangential displacement at the contact point of the bodies.

Criteria of pure rolling: $u_S \neq 0$ and $u_A = 0$.