Széchenyi István University

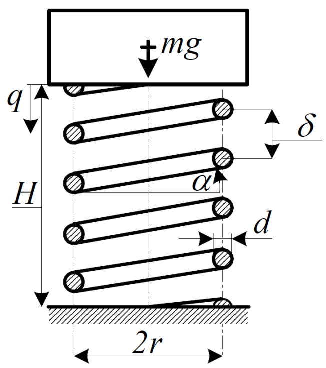
Department of Applied Mechanics

Dynamics of Machines	Theoretical questions for MSc students in Vehicle Engineering and MSc students in Mechanical Engineering

February 27, 2020

1. How is the center of mass of a rigid body defined on the base of definition of linear momentum of body with a continuous mass distribution? How is the position vector of the center of mass calculated by means of the mass and static moment of the body about any point? Justify your answer.
2. Derive the theorem of linear momentum about any reference point for a rigid body on the base of definition of linear momentum of body with a continuous mass distribution. Formulate the theorem of linear momentum for pure translational motion and for case if the reference point is the center of mass.
3. How is moment of inertia tensor about the center of mass of a rigid body defined on the base of definition of angular momentum of body with a continuous mass distribution?
4. Derive the theorem of angular momentum about any reference point for a rigid body on the base of definition of angular momentum of body with a continuous mass distribution. Formulate the theorem of angular momentum for a stationary point and for case if the reference point is the center of mass.
5. Derive the kinetic energy of a rigid body about any reference point on the base of definition of kinetic energy of body with a continuous mass distribution. Formulate the kinetic energy for pure translational motion, for a stationary point and for case if the reference point is the center of mass.
6. Derive the power of force system acting on a rigid body about any reference point on the base of definition of power of force system.
7. Write down the matrix of moment of inertia tensor of rigid body in Cartesian coordinate system. Write down the calculation method of each entry of the matrix.

8.

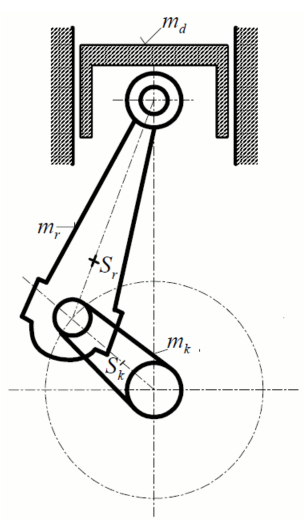


Calculate the spring constant c of spring in the figure if length of the wire l, diameter of the wire d, radius of the spring r, pitch of the spring δ, pitch angle α, shear modulus G, Poisson's ratio ν, compression q of the spring under load mg, and the unloaded length of the spring H are known.

9. List the components of a vibrating system.

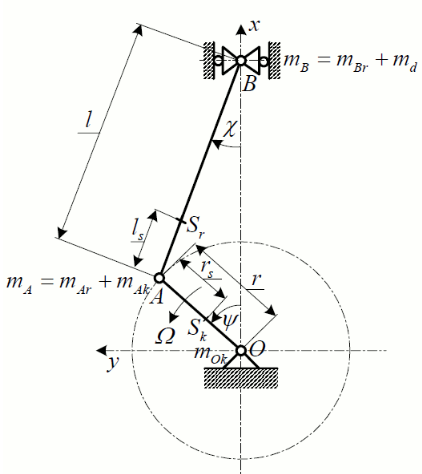
10. Derive the relation between Lehr's damping and the logarithmic decrement by means of the equation of motion of a damped, non-excited vibrating system with one degree of freedom.

11.



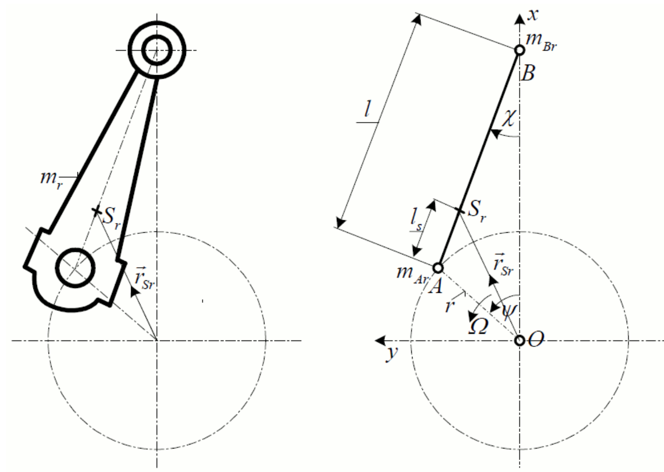
Geometrical dimensions and masses of the parts of the crank mechanism seen in the figure are known. (The tensor of inertia and the positions of the centers of mass and are known, too.) Sketch a replacement model in which the masses of the parts are reduced into the joints of rods replacing the parts. Determine the value of these masses as function of geometrical dimensions. Denote every used quantity in the figure.

12.



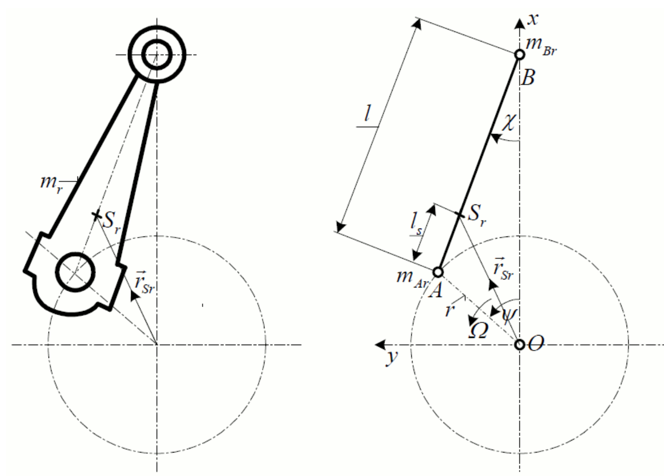
The dimensions of the mechanism are shown in the figure (and the center of mass of the components). The masses reduced to points A, B, and O are given. Calculate the unbalanced forces acting on the point O at constant angular velocity Ω of the connecting rod OA. Expand the forces in series of the harmonics of the angular velocity. Ignore any terms of higher degree than quadratic.

13.



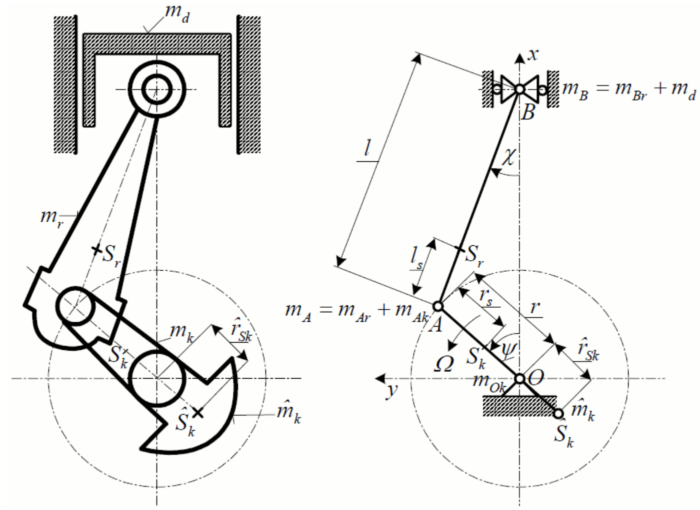
The geometrical dimensions and masses of the components of the crank mechanism shown in the figure are known (including the centers of mass and the tensors of inertia). Using the original structure and the replacement model, write down the unbalanced moment by means of the angular acceleration $\ddot{\chi}$ of the connecting rod. How can the moment of inertia of the replacement model about axis z be determined?

14.



The dimensions of the mechanism shown in the figure and the centers of mass of the components are given. The moments of inertia of the original structure J_{Srz} and the replacement model $\tilde{J}_{Srz}$ are known, too. $M_{Oz} = \left(\tilde{J}_{Srz} - J_{Srz}\right)\ddot{\chi}$ is the unbalanced moment at constant angular velocity Ω of the crank. Expand the unbalanced moment in series of the harmonics of the angular velocity. Ignore any terms of higher degree than quadratic.

15.

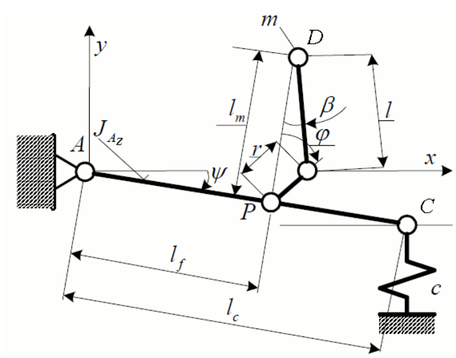


The dimensions of the mechanism shown in the figure (and the centers of mass of the components) as well as the masses reduced to points A, B and O and the balancing mass are known. Write down the x and y coordinates of the unbalanced force taking into account the balancing mass. Knowing this relationship, what are the possibilities for balancing the forces?

16. Draw a mechanism for balancing the forces of a single-cylinder crank mechanism so that the fundamental and the second harmonic frequencies of the forces disappear. Write down the relationship between the constants in the equation of the unbalanced forces and the masses and dimensions of members of the additional mechanism.

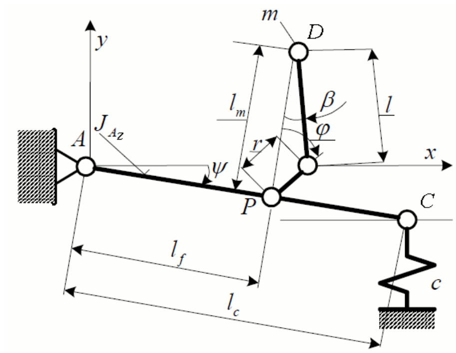
17. How can the balancing of moments of a crank mechanism be done?

18.



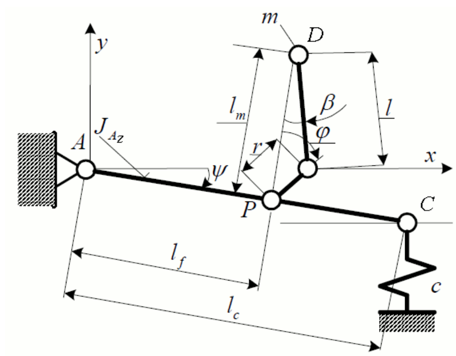
Give the position of point D (piston) as a function of time in the coordinate system xy. The crank r revolves with constant angular velocity Ω around point P.

19.



Calculate the kinetic energy of the structure shown in the figure if the mass of the piston D, the moment of inertia J_{Az} of the rod AC, and the distance l_f are known. Consider l_m and angle ψ as unknown functions of time.

20.



Derive the equation of motion of the structure in the figure if the kinetic energy

$$E = \frac{1}{2} J_{Az} \dot{\psi}^2 + \frac{1}{2} m \left[\left(l_f \dot{\psi} - \dot{l}_m \right)^2 + \left(l_m \dot{\psi} \right)^2 \right]$$

and the spring constant c are known. The unknown function of time in the equation of motion is the angle ψ.

21. The equation of motion of parametrically excited vibration is known:

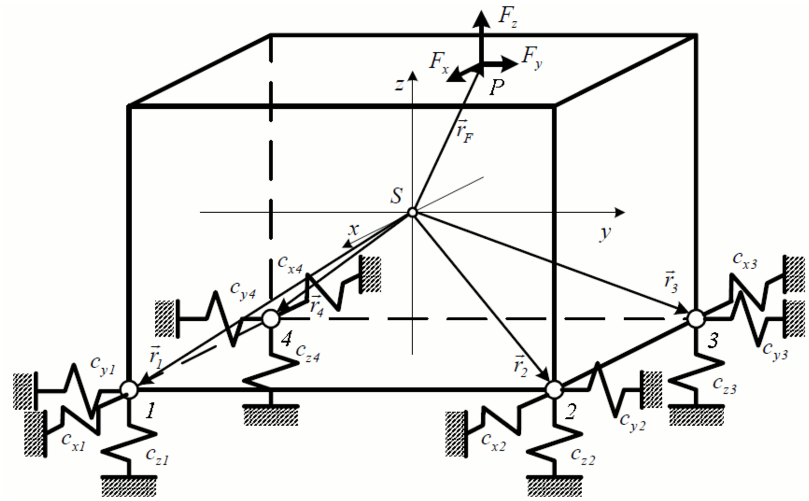
$$\left[J_{Az} + m \left(l_f^2 + (l + r\cos(\Omega t))^2 \right) \right] \ddot{\psi} - 2r\Omega m \sin(\Omega t) (l + r\cos(\Omega t)) \dot{\psi} + \frac{l_c^2 \psi}{c} = -m l_f r \Omega^2 \cos(\Omega t).$$

Calculate the averages of the coefficients over a period.

22. Calculate the particular solution of differential equation describing vibrations of a structure:

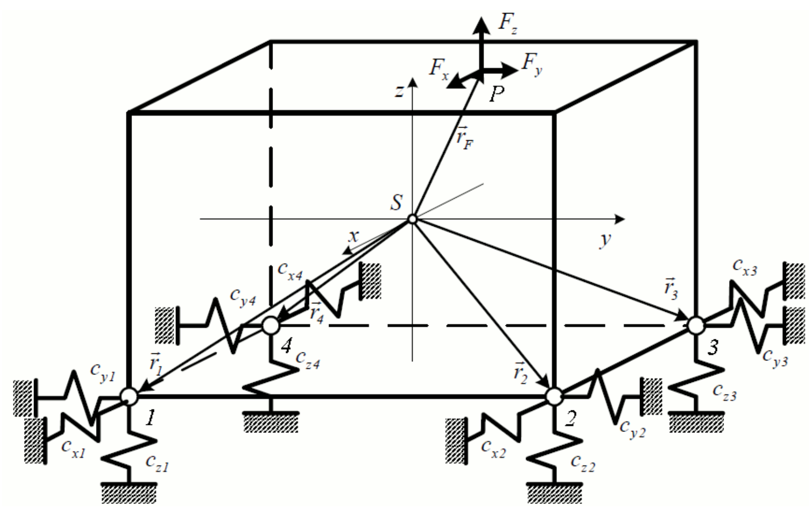
$$\ddot{y}_P + \frac{l_c^2}{J_{red} c} y_P = -\frac{m l_f^2 r}{J_{red}} \Omega^2 \cos(\Omega t).$$

23.



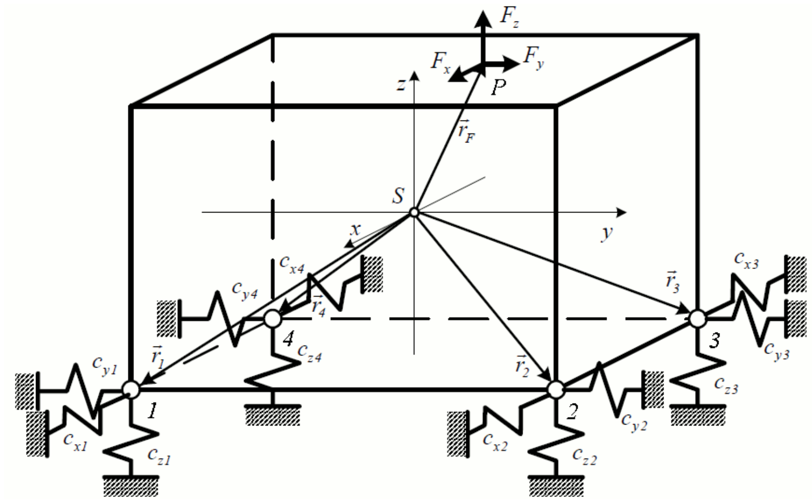
The size, support, mass, moment of inertia and load of the machine base shown in the figure are given. Position vectors pointing from the center of mass to points 1, 2, 3, 4, and P are known. Write down the displacements and velocities of points 1, 2, 3, 4, and P in vector and matrix form using the displacement $\vec{u}_S$ of the center of mass and the (rigid-body-like) rotation angle $\vec{\varphi}$ of the machine base.

24.



Write down the strain energy of the springs supporting the machine base in matrix form. The displacements of the corners 1, 2, 3, 4 in directions x, y, and z are known: u_i, v_i, and w_i. Also the spring constants c_{x1}, c_{yi}, and c_{zi} are known. $(i = 1, 2, 3, 4)$

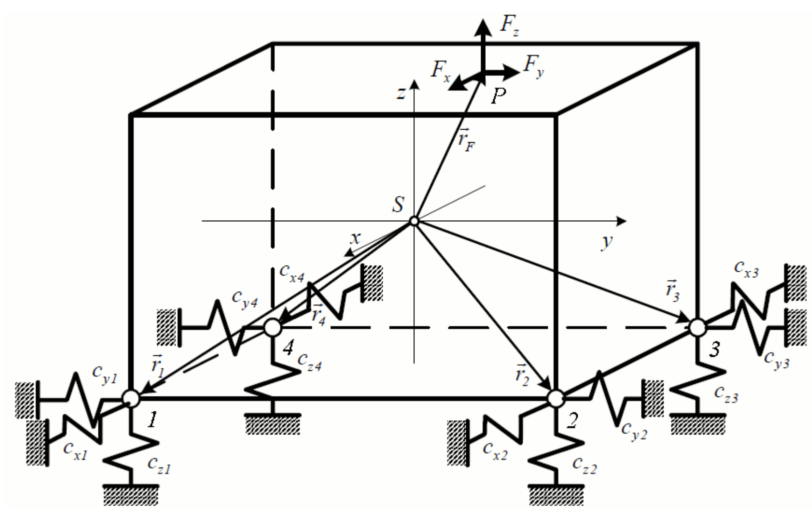
25.



Dimensions ($\vec{r}_i$), displacements of center of mass ($\vec{u}_S$), and (rigid-body-like) rotation angles ($\vec{\varphi}$) of a machine base are given. Spring constants (c_{x1}, c_{yi}, c_{zi}) are also known. How can the deformation energy be expressed by means of these quantities (e.g. using the $\underline{\underline{q}}^T = [u_S \;\; v_S \;\; w_S \;\; \varphi_x \;\; \varphi_y \;\; \varphi_z]$ matrix) and the following relation?

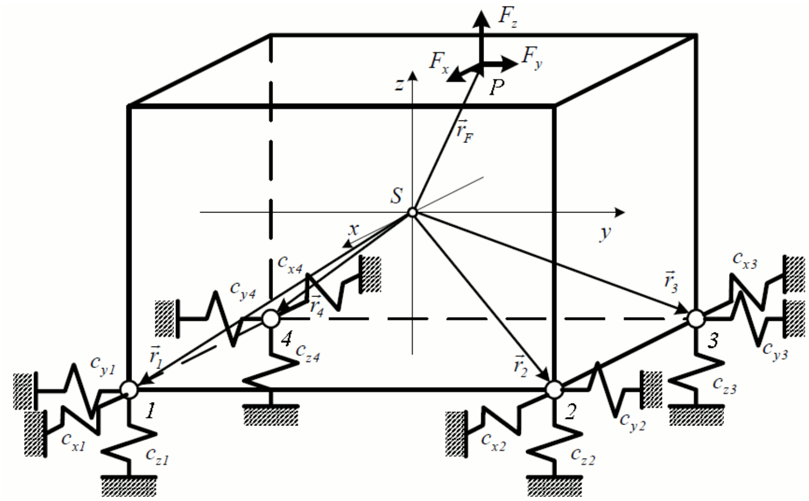
$$U = \sum_{i=1}^{4} \frac{1}{2} \begin{bmatrix} u_i & v_i & w_i \end{bmatrix} \begin{bmatrix} \frac{1}{c_{xi}} & 0 & 0 \\ 0 & \frac{1}{c_{yi}} & 0 \\ 0 & 0 & \frac{1}{c_{zi}} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \sum_{i=1}^{4} \frac{1}{2} \underline{\underline{u}}_i^T \underline{\underline{C}}_i \underline{\underline{u}}_i .$$

26.



Position of the center of mass ($\vec{u}_S$), rigid body-like rotation angle ($\vec{\varphi}$), mass (m), and moment of inertia ($\underline{\underline{J}}_S$) of machine base shown in the figure are known. Write down the kinetic energy of the machine base with vector and matrix notation. In case of matrix form the displacement of the machine base let matrix $\underline{\underline{q}}^T = \begin{bmatrix} u_S & v_S & w_S & \varphi_x & \varphi_y & \varphi_z \end{bmatrix}$ be.

27.



The strain energy of the elastic embedding ($U = \frac{1}{2}\underline{\underline{q}}^T \underline{\underline{\underline{C}}}\,\underline{\underline{q}}$) and the kinetic energy ($E = \frac{1}{2}\underline{\underline{\dot{q}}}^T \underline{\underline{\underline{M}}}\,\underline{\underline{\dot{q}}}$) of the machine base are given where $\underline{\underline{q}}^T = \begin{bmatrix} u_S & v_S & w_S & \varphi_x & \varphi_y & \varphi_z \end{bmatrix}$. The load on the machine base is given by matrix $\underline{\underline{F}}^T = \begin{bmatrix} F_x & F_y & F_z \end{bmatrix}$. Derive the equation of motion of the machine base.

28. The equation of motion $\underline{\underline{\underline{M}}}\,\underline{\underline{\ddot{q}}} + \underline{\underline{\underline{C}}}\,\underline{\underline{q}} = \underline{\underline{0}}$ for a spatial (3D) machine base without excitation is given. Calculate the function $\underline{\underline{q}}(t)$ describing the eigen-vibrations of the machine base. Specify the quantities obtained during the calculation.

29. The equation of motion of a spatial (3D) machine base with excited vibrations is given by $\underline{\underline{\underline{M}}}\,\underline{\underline{\ddot{q}}} + \underline{\underline{\underline{C}}}\,\underline{\underline{q}} = \underline{\underline{Q}}_F$. Write down the solution of the equation of motion: $\underline{\underline{q}}(t)$. How are the quantities in the solution calculated?

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30. Under what conditions can a rotating structure be called a Laval rotor?

31. Derive the equation of motion of a Laval rotor. Draw a figure for depiction of the quantities used.

32. Solve the equation of motion of a Laval rotor assuming that there is alway some extent of dissipation.

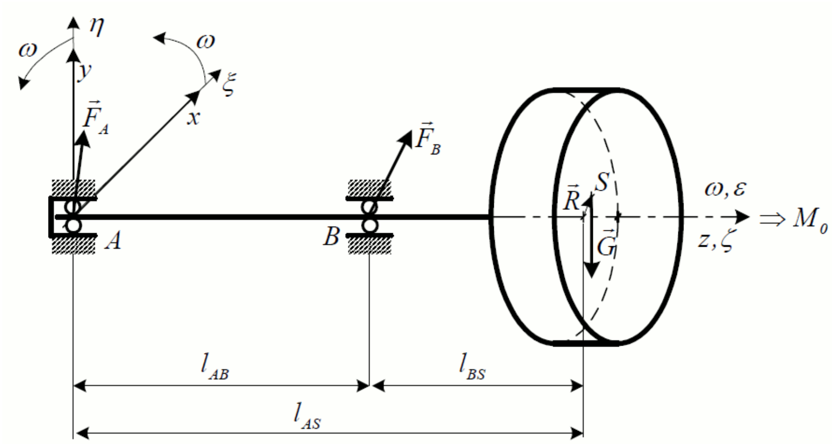
$$m\left(\ddot{x} - e\Omega^2 \cos(\Omega t)\right) = -\frac{x}{c}$$
$$m\left(\ddot{y} - e\Omega^2 \sin(\Omega t)\right) = -\frac{y}{c}$$

Here m is the mass of the disk, e is the eccentricity, Ω is the angular velocity of the rotor, and c is the spring constant generated by the bending of the shaft.

33. A Laval rotor is being rotated with constant angular velocity $\Omega = \Omega_{krit}$. Its centerline is at rest initially. Calculate the amplitude of vibrations of the center of the rotor as function of time t if the motion of the center in plane xy is described by the following system of equations

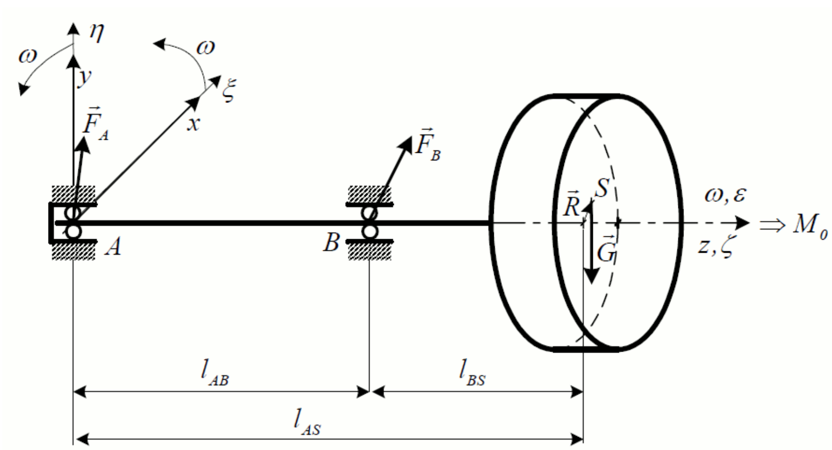
$$\left.\begin{aligned} \ddot{x} + \alpha^2 x = e\Omega^2 \cos(\Omega t) \\ \ddot{y} + \alpha^2 y = e\Omega^2 \sin(\Omega t) \end{aligned}\right\}.$$

34.



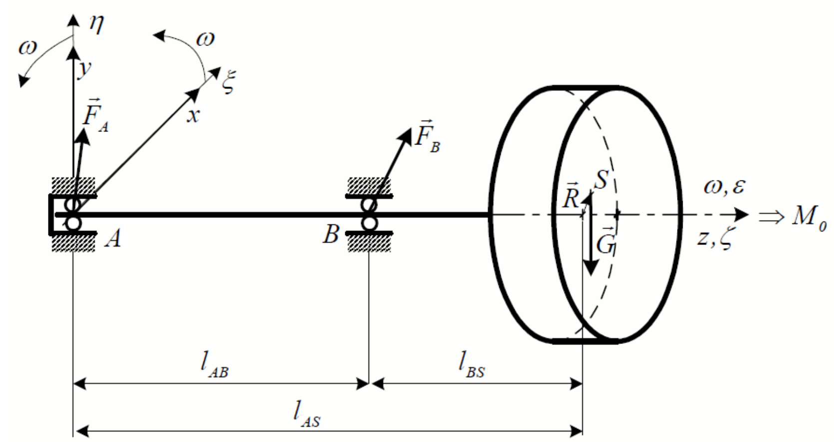
There is a statically and dynamically unbalanced wheel in the figure. The shaft is supported by bearings in point A and B. Dimensions and loads of the structure are given. Write down the theorem of linear momentum for the structure and the theorem of angular moment about point A. Give the coordinates of the vector quantities of the equations.

35.



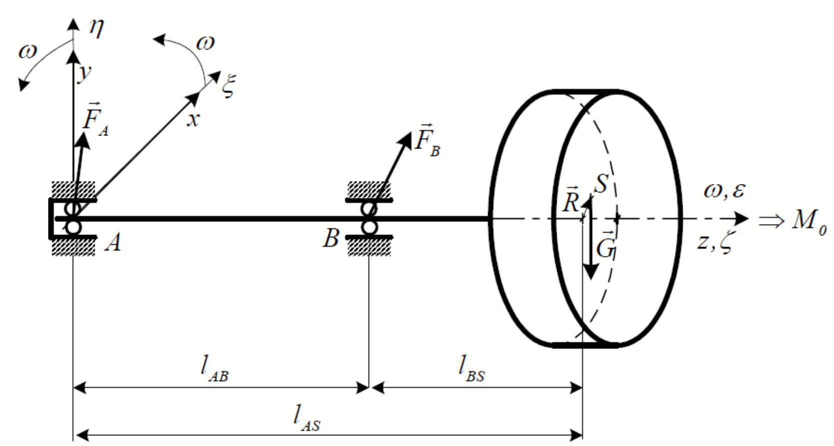
There is a statically and dynamically unbalanced wheel in the figure. The shaft is supported by bearings in point A and B. Dimensions and loads of the structure are given. Write down the theorem of linear momentum for the structure and the theorem of angular moment about point B. Give the coordinates of the vector quantities of the equations.

36.



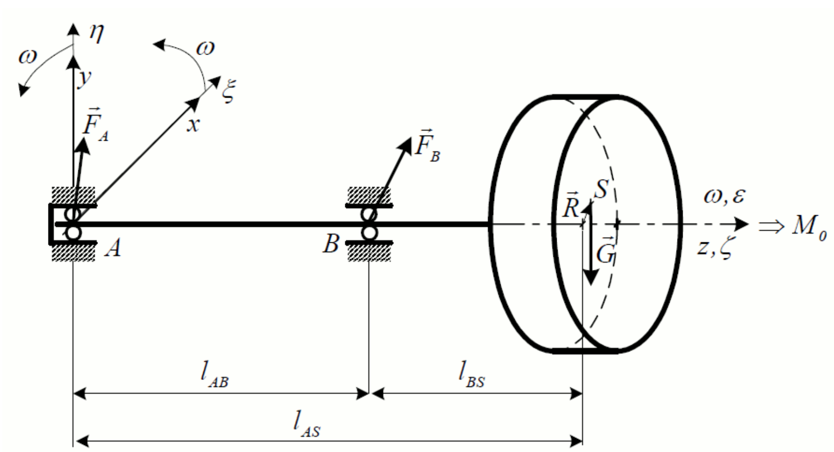
What does it mean that the structure shown is statically unbalanced? Justify your answer using the theorem of linear momentum!

37.



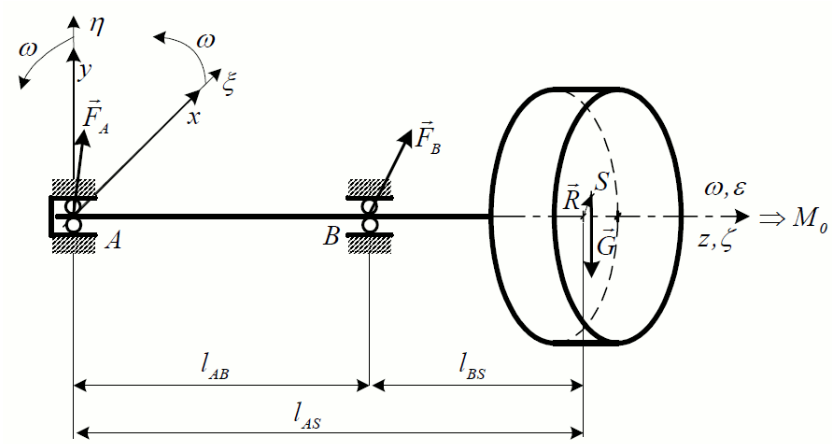
A moment M_0 = constant is acting on the unbalanced disk shown in the figure. What can be said about the angular acceleration of the disk? Justify your answer.

38.



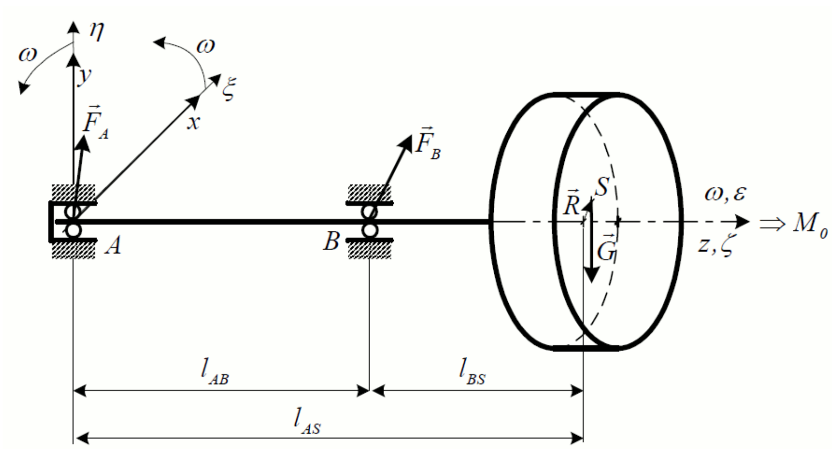
Calculate support force $\vec{F}_A$ of the structure shown in the figure by means of the theorem of angular momentum.

39.



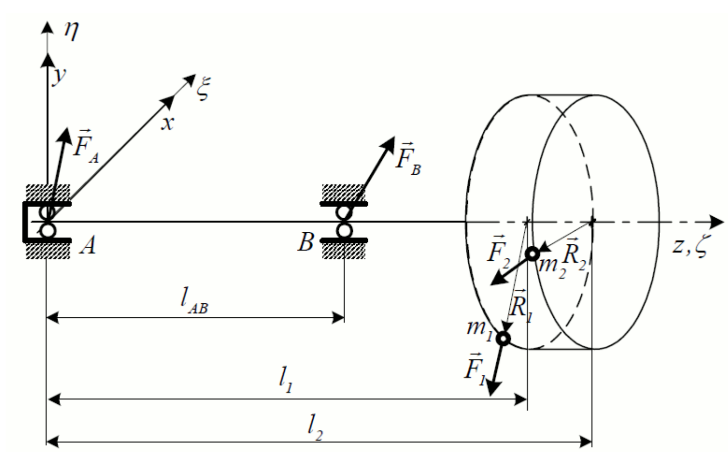
Calculate support force $\vec{F}_B$ of the structure shown in the figure by means of the theorem of angular momentum.

40.



A statically and dynamically unbalanced disk is shown in the figure. How can the structure be balanced? Write down the equations for balancing. How many equations and unknowns are there?

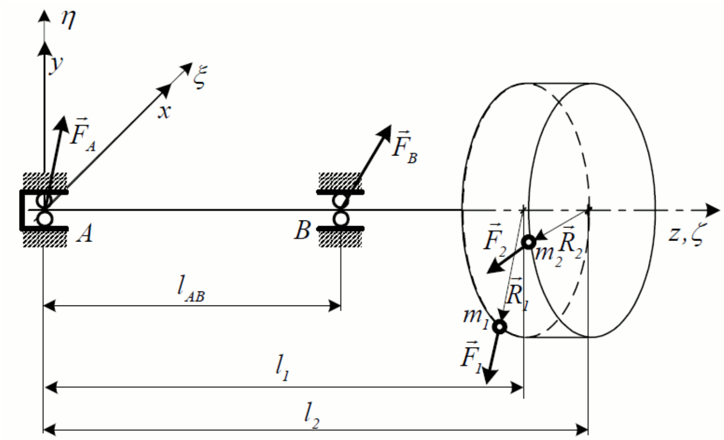
41.



A statically and dynamically unbalanced disk is modeled by placing two point masses on a balanced disk (see figure). Force $\vec{F}_A$, angle α between force $\vec{F}_A$ and axis x, force $\vec{F}_B$, angle

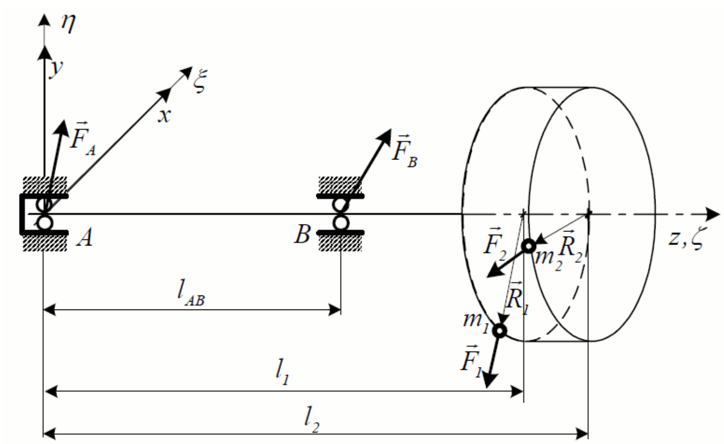
β between force $\vec{F}_B$ and axis x, and the dimensions are known. Calculate mass m_1 using known quantities.

42.



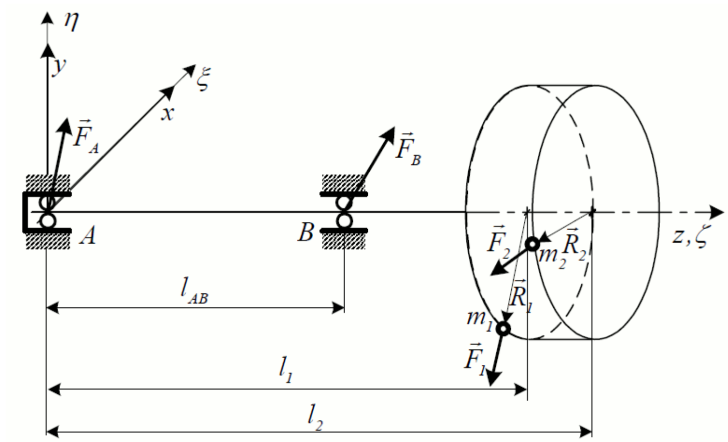
A statically and dynamically unbalanced disk is modeled by placing two point masses on a balanced disk (see figure). Force $\vec{F}_A$, angle α between force $\vec{F}_A$ and axis x, force $\vec{F}_B$, angle β between force $\vec{F}_B$ and axis x, and the dimensions are known. Calculate mass m_2 using known quantities.

43.



A statically and dynamically unbalanced disk is modeled by placing two point masses on a balanced disk (see figure). Force $\vec{F}_A$, angle α between force $\vec{F}_A$ and axis x, force $\vec{F}_B$, angle β between force $\vec{F}_B$ and axis x, and the dimensions are known. Calculate angle γ between force $\vec{F}_1$ and axis x using known quantities.

44.



A statically and dynamically unbalanced disk is modeled by placing two point masses on a balanced disk (see figure). Force $\vec{F}_A$, angle α between force $\vec{F}_A$ and axis x, force $\vec{F}_B$, angle β between force $\vec{F}_B$ and axis x, and the dimensions are known. Calculate angle δ between force $\vec{F}_2$ and axis x using known quantities.

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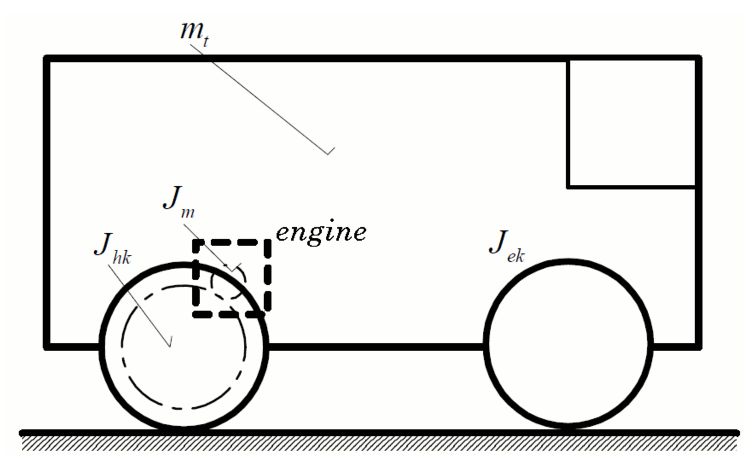
45. What are Wittenbauer's first and second basic problems?

46. Derive the equation of motion of a machine that can be modeled as a mechanism with one degree of freedom (Eksergian's equation). Draw a figure with the applied notations.

47. Describe the solution of the equation of motion (Eksergian's equation) of a machine that can be modeled as a mechanism with one degree of freedom for a conservative case ($Q = Q(\varphi)$).

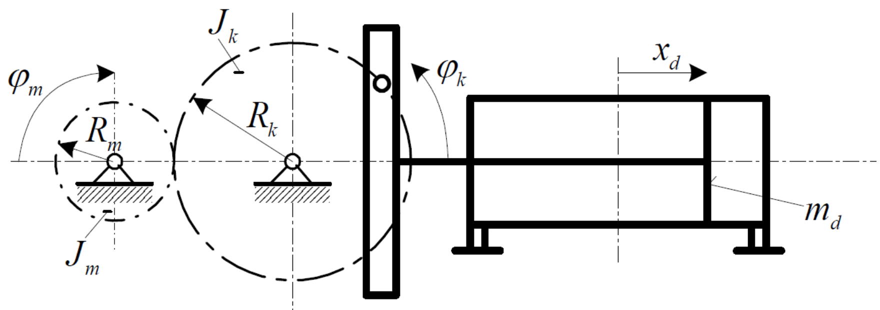
48. Calculate the average speed $\dot{\varphi}_a$ of a machine modeled as a single-degree mechanism with a constant drive if the initial kinetic energy of the mechanism is much greater than the work of external forces.

49.



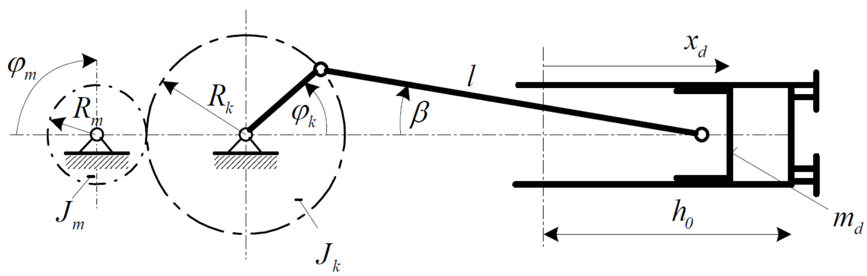
The following quantities of the electric vehicle shown in the figure are given: mass m, moment of inertia J_{ek} and J_{hk} of the front and rear wheels, radii R_k of the wheels, and moment of inertia J_m of the rotor of the engine. The characteristics of the engine driving the vehicle is $M(\dot{\varphi}) = M_{max}\left(1 - \frac{\dot{\varphi}}{\dot{\varphi}_{max}}\right)$. Gear ratio n between motor and rear wheel is given. Derive the equation of motion of the vehicle excluding the running and rolling resistance. (The wheels roll without slipping.) Solve the equation of motion for a vehicle started from rest ($\varphi_0 = 0$, $\dot{\varphi}_0 = 0$). Plot angular rotation φ, angular velocity $\dot{\varphi}$, and angular acceleration $\ddot{\varphi}$ as functions of time t.

50.



A coulisse mechanism is shown in the figure. The combined moment of inertia of the engine and the gear is J_m. The radii of the gears are R_m and R_k, their angular rotations are φ_m and φ_k. The mass of the piston is m_d. The displacement of the piston is x_d. Derive the equation of motion of the mechanism if the characteristic of the engine is $M(\dot{\varphi}) = M_{max}\left(1 - \frac{\dot{\varphi}}{\dot{\varphi}_{max}}\right)$.

51.



A crank mechanism is shown in the figure. The combined moment of inertia of the engine and the gear is J_m. The radii of the gears are R_m and R_k, their angular rotations are φ_m and φ_k. The length of the connecting rod is l. The mass, the displacement, and the area of the piston are m_d, x_d, A_d, respectively. The length of the cylinder is h_0. The pressure p_d acting on the piston can be determined from the relation $p_d V = p_0 V_0$ (Boyle-Mariotte gas law) where V is the instantaneous volume of the cylinder, p_0 is the initial pressure and V_0 is the initial volume of the gas. Derive the equation of motion of the mechanism if the characteristic of the engine is $M(\dot{\varphi}) = M_{max}\left(1 - \frac{\dot{\varphi}}{\dot{\varphi}_{max}}\right)$.