SZÉCHENYI ISTVÁN UNIVERSITY | **DEPARTMENT OF APPLIED MECHANICS**

MECHANICS – STRENGTH OF MATERIALS	**Theoretical questions and answers for BSc students**

(1) What is the subject of strength of materials?

The subject of strength of materials is the description of kinematics, dynamics, and material structure behaviour of deformable bodies which are in permanent rest before and after load.

(2) What is load?

The effect of bodies not belonging to the system studied. Its magnitude is known.
Load $\equiv$ know outer force system.

(3) Define the concept of strain.

Strain means that the points of a body move such way under load that the geometry (e.g. length, angle, surface, volume) of the body change.

(4) What does kinematics mean in strength of materials?

Kinematics describes the strains and displacements of points of body caused by load.

(5) What does dynamics mean in strength of materials?

Dynamics describes the internal force system caused by load.

(6) What does behaviour of structure of materials mean in strength of materials?

The behaviour of structure of materials determines the relation between strain and internal force system.

(7) Define the concept of a body model.

A body model has idealized features which reflect the most important features of real bodies from point of view of the study.

(8) Define the concept of a rigid body.

It is body model in which the distance between any two points is constant. (The distance between points does not change under load.)

(9) Define the concept of a solid body.

There can be deformations in this body model. (The distance between points can change under load.)

(10) What characterizes the rigid-body-like motion? What kind of rigid-body-like motions exist?

In the case of rigid-body-like motion points of a body move without changing their distances.
Rigid-body-like motion: - rigid-body-like translation,
- rigid-body-like rotation.

(11) What do elastic and plastic strain (deformation) mean?

- Strain is elastic if the body regains its original shape after removing the load acting on it.
- Strain is plastic if the body does not regain its original shape after removing the load acting on it.

(12) What do small displacements and strains (deformations) mean?

- In the case of small displacements the displacement of the points of the body is orders of magnitude smaller than the characteristic geometrical size of the body.
- In the case of small strains the characteristic quantities of strain are essentially smaller than one: $\varepsilon << 1$, $\gamma << 1$.

(13) What is the geometrical meaning of the normal strain ε_y? How is the sign of the normal strain defined?

ε_y - changing of unit length in direction *y* under load.

$\varepsilon_y > 0$ - extension, stretching, $\varepsilon_y < 0$ - compression, shortening.

(14) Give the geometrical meaning of the shear strain γ_{xz} and its sign convention.

γ_{xz} - changing of angle of 90° between directions *x* and *z* under load.

$\gamma_{xz} > 0$ - angle of 90° decreases, $\gamma_{xz} < 0$ - angle of 90° increases.

(15) Write down the dyadic and matrix form of the strain tensor in point P.

- dyadic form: $\underline{\underline{A}}_P = (\vec{\alpha}_x \circ \vec{i} + \vec{\alpha}_y \circ \vec{j} + \vec{\alpha}_z \circ \vec{k})$

where $\vec{\alpha}_x$, $\vec{\alpha}_y$, and $\vec{\alpha}_z$ are the strain vectors belonging to directions *x, y,* and *z*, respectively.

- matrix form: $\left[\underline{\underline{A}}_P\right] = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z \end{bmatrix}$.

(16) Give definition, notation and SI unit of the stress vector.

Definition 1: Stress vector is the density vector (intensity vector) of the internal force system distributed over surface under load. Sign: $\vec{\rho}_n$.

Definition 2: $\vec{\rho}_n = \dfrac{d\vec{F}_b}{dA}$ - stress vector is the internal force of body over a unit surface under load.

Its unit in International System of Units: $N/m^2 = Pa$ (Pascal).

(17) What are the names and physical meaning of the stress coordinates σ_n and τ_{mn}?

σ_n is the normal stress. It is the coordinate along direction $\vec{n}$ of the stress vector on the elementary surface with normal vector $\vec{n}$.
τ_{mn} is the shear stress. It is the coordinate along direction $\vec{m}$ of the stress vector on the elementary surface with normal vector $\vec{n}$.

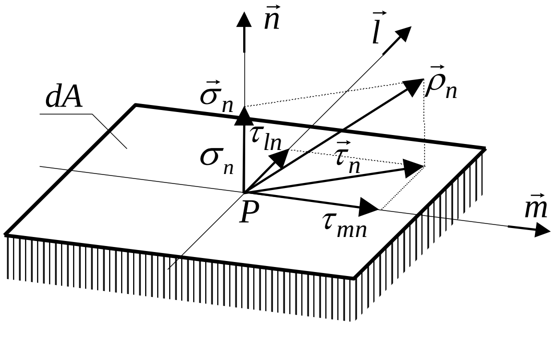
(18) Write down the dyadic and matrix form of the stress tensor in point P.

- dyadic form: $\underline{\underline{F}}_P = (\vec{\rho}_x \circ \vec{i} + \vec{\rho}_y \circ \vec{j} + \vec{\rho}_z \circ \vec{k})$

  where $\vec{\rho}_x$, $\vec{\rho}_y$, and $\vec{\rho}_z$ are the stress vectors on surfaces perpendicular to directions *x*, *y*, and *z*, respectively.

- matrix form: $\left[\underline{\underline{F}}_P\right] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$.

(19) How can stress vector $\vec{\rho}_n$ (belonging to plane with normal unit vector $\vec{n}$) and its coordinates be calculated knowing the stress tensor?



- stress vector: $\vec{\rho}_n = \underline{\underline{F}}_P \cdot \vec{n}$.
- normal stress: $\sigma_n = \vec{n} \cdot \vec{\rho}_n = \vec{n} \cdot \vec{\sigma}_n$.
- shear stresses: $\tau_{mn} = \vec{m} \cdot \vec{\rho}_n = \vec{m} \cdot \vec{\tau}_n$,

$$\tau_{ln} = \vec{l} \cdot \vec{\rho}_n = \vec{l} \cdot \vec{\tau}_n.$$

(20) What are the definitions of a rod and its cross section?

Rod: a body whose one dimension is much greater than the other two.
Cross section: it is a section which is perpendicular to the largest dimension of the rod.

(21) Define the centerline of a rod. What is the mechanical model of a rod?

Centerline is formed from the the centers of gravity of the cross sections of the rod.
Centerline is the mechanical model of a rod.

(22) Define the concept of a prismatic rod.

Definition 1: If the size and shape of the cross sections along the length of the rod do not change.

Definition 2: If the cross sections of the rod can be shifted parallelly into each other along the centerline.

(23) Define the internal forces of a rod.

There is a distributed load in any cross section of the rod. Internal forces are the equivalent force couple system (or its coordinates) of this load about the center of gravity of the cross section.

(24) Define the pure tension and compression.

Pure tension and compression: the internal forces of every cross section of the rod are only (exclusively) normal forces.

(25) Write down the stress tensor in point P in the case of pure tension or compression.

$$\left[\underline{\underline{F}}_P\right] = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \sigma_x = \frac{N}{A} = \text{ constant.}$$

Where A is the area of the cross section and N is the normal force.

(26) Write down the strain tensor in point P in the case of pure tension or compression.

$$\left[\underline{\underline{A}}_P\right] = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

Longitudinal elongation: $\varepsilon_x = \dfrac{l'-l}{l} = \text{constant.}$

Transverse elongations: $\varepsilon_k = \varepsilon_y = \varepsilon_z = -\nu\,\varepsilon_x = \text{constant.}$

l – length of the rod without load, l' – deformed length of the rod,
ν – Poisson's ratio (material property).

(27) How can the strain energy density and the total strain energy be calculated in a rod in the case of pure tension or compression?

- strain energy density: $u = \dfrac{1}{2}\varepsilon_x\,\sigma_x$.
- total (in the whole rod) strain energy: $U = \displaystyle\int_{(V)} u\, dV = \frac{1}{2}\frac{N^2}{AE}l$,

$V = Al$ - volume of the rod.

(28) Define the uniaxial stress state. When does it occur?

A stress state at a point P is uniaxial if only one element in the stress tensor is different from zero and this non-zero element is in the main diagonal.
Uniaxial stress state occurs under tension/compression and bending.

(29) Write down the simple Hooke's law for tension and compression.

$\sigma_x = E\varepsilon_x$ and $\varepsilon_z = \varepsilon_y = \varepsilon_k = -\nu\varepsilon_x$,

where: σ_x normal stress along the rod, ε_x normal strain along the rod,

$\varepsilon_z = \varepsilon_y = \varepsilon_k$ transverse specific elongation,

E Young's modulus, ν Poisson's ratio.

(30) What is the goal of sizing in the case of rod structures?

Given: material and load of the rod.
Task: determine the dimensions of the cross sections of the rod so that the rod can bear the load safely.

(31) What is the goal of check in the case of rod structures?

Given: material, dimensions of cross sections, and load of the rod.
Task: decide whether the rod can bear the load safely.
- If it does, the rod is applicable.
- If it does not, the rod is not applicable.

(32) What are the pure bending and homogeneous internal forces?

Pure bending: if the internal force system of the cross sections of the rod comprimes only of bending moment.
Homogeneous internal forces (bending): if the internal force system (bending moment) does not change along the rod.

(33) Explain the Euler–Bernoulli beam theory.

In the case of pure homogeneous bending the cross sections of the rod remain planes and perpendicular to the deformed centerline of the rod. There is not any shear strain in the plane of the cross section.

(34) Write down the stress tensor in point P of a rod (beam) in the case of pure bending.

$$\left[\underline{\underline{F}}_P\right] = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \sigma_x = \sigma_x(y) = \frac{M_{hz}}{I_z} y \,.$$

Where M_{bz} is the bending moment, I_z second moment of area of the cross section with respect to axis z, y is the coordinate of the point where the stress is to be determined.

(35) Write down the strain tensor in point P of a rod (beam) in the case of pure bending.

$$\left[\underline{\underline{A}}_P\right] = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

Longitudinal elongation: $\varepsilon_x = \kappa y = \dfrac{1}{R} y = \dfrac{\sigma_x}{E}$.

Transverse elongations: $\varepsilon_k = \varepsilon_y = \varepsilon_z = -\nu\,\varepsilon_x$

κ curvature of the centerline, R radius of curvature of the centerline,

E Young's modulus, ν Poisson's ratio.

(36) Define the concept of straight bending.

If bending moment vector $\vec{M}_b$ is parallel with one of the principal axes of the cross section about the center of mass.

(37) How can the maximum stress be calculated in the case of pure straight bending? Define the dangerous point of the cross section.

$$\sigma_{x\max} = \frac{|M_{bz}|}{I_z}|e_{\max}| = \frac{|M_{bz}|}{K_z},$$

$e_{\max}$ y coordinate of point whose distance from axis z is maximal in the cross section,

K_z section modulus of the cross section.

Dangerous point: point of the cross section where $\sigma_{x\max}$ occurs.

(38) How can the strain energy density and the total strain energy of a rod be calculated in the case of pure straight bending?

- strain energy density: $u = \frac{1}{2}\varepsilon_x \sigma_x$.
- total (in the whole rod) strain energy: $U = \int_{(V)} u\, dV = \frac{1}{2}\frac{M_{hz}^{\;2}}{I_z E} l$.

V is the volume of the rod, l is the length of the rod, M_{bz} is the bending moment, I_z is the planar second moment of area with respect to axis z, E is Young's modulus.

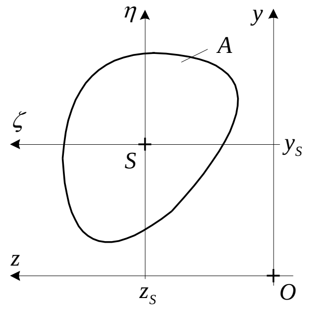
(39) What are the planar second moment, product moment and polar second moment of area?

$I_z = \int_{(A)} y^2 dA > 0$, $I_y = \int_{(A)} z^2 dA > 0$ - planar second moments of area with respect to axis z and y,

$I_{yz} = I_{zy} = \int_{(A)} y\, z\, dA = \int_{(A)} z\, y\, dA$ - product moments of area with respect to plane yz.

$I_p = \int_{(A)} r^2 dA = \int_{(A)} (x^2 + y^2)\, dA > 0$ - polar second moment of area.

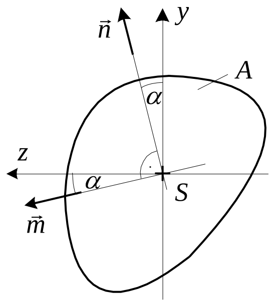
(40) Explain Steiner's theorem.



$$I_y = I_\eta + A z_S^2,$$
$$I_z = I_\zeta + A y_S^2,$$
$$I_{yz} = I_{\eta\zeta} + A z_S y_S .$$

S is the center of gravity of the area. Steiner's theorem gives relationship between the second moments of area (moments of inertia).

(41) Tensor of second moment of area is known in coordinate system xy whose origin is the center of mass (point S) of the cross section. How can one calculate the second moments of area with respect to axes $\vec{n}$, $\vec{m}$ about point S?



$$\underset{=S}{I} = \begin{bmatrix} I_y & -I_{yz} \\ -I_{zy} & I_z \end{bmatrix}$$

$$I_n = \vec{n} \cdot \underset{=S}{I} \cdot \vec{n} \ ,$$
$$I_m = \vec{m} \cdot \underset{=S}{I} \cdot \vec{m} \ .$$
$$I_{nm} = I_{mn} = -\vec{n} \cdot \underset{=S}{I} \cdot \vec{m} = -\vec{m} \cdot \underset{=S}{I} \cdot \vec{n}$$

(42) Give definition of principal axes and principal second moments of area of a cross section.

Axes *1* and *2* are called principal axes if the product moments of area with respect to plane *1-2* are zeros: $I_{12} = I_{21} = 0$. Axes *1* and *2* are always perpendicular to each other.

Principal second moments of area are the planar second moments of area with respect to axis *1* and *2*.

(43) What are the most important theorems of the principal axes?

- There exists at least one couple of principal axes for any cross section. These axes are perpendicular to each other.
- A symmetry axis of a cross section is always a principal axis. An axis perpendicular to this symmetry axis is always also principal axis.
- If there are more than two principal axes about center of mass, any axis which passes through the center of mass is principal axis. In this case $I = I_1 = I_2$.

(44) Define the pure torsion. What kind of cross sections do we deal in the case of pure torsion with?

Pure torsion: the internal forces of every cross section of the rod are only (exclusively) torsions.

We deal with only circular and circular ring cross sections in the case of pure torsion.

(45) What is the matrix of the stress of circular and circular ring cross sections in point P in cylindrical coordinate system in the case of pure torsion?

$$\underset{(R\varphi x)}{\left[\underset{=P}{F}\right]} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \tau_{\varphi x} \\ 0 & \tau_{x\varphi} & 0 \end{bmatrix}, \qquad \tau_{x\varphi}(R) = \tau_{\varphi x}(R) = \frac{M_t}{I_p} R \, .$$

Here M_t is the torsion moment, I_p is the polar second moment of area of the cross section of the rod, R is the coordinate of point where the stress is to be determined.

(46) What is the matrix of the strain of circular and circular ring cross sections in point P in cylindrical coordinate system in the case of pure torsion?

$$\underset{(R\varphi x)}{\left[\underline{\underline{A}}\right]} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\gamma_{\varphi x} \\ 0 & \frac{1}{2}\gamma_{x\varphi} & 0 \end{bmatrix}$$

$\gamma_{\varphi x}(R) = \gamma_{x\varphi}(R) = \vartheta R = \dfrac{\tau_{\varphi x}}{G}$

R is the coordinate of point where the strain is to be determined, ϑ =constant – is the shear strain, G is the shear modulus.

(47) How can the strain energy density and the total strain energy be calculated in a rod with circle and circle ring cross section in the case of pure torsion?

- strain energy density: $u = \dfrac{1}{2}\gamma_{x\varphi}\,\tau_{x\varphi}$.
- total strain energy in the rod: $U = \displaystyle\int_{(V)} u\,dV = \frac{1}{2}\frac{M_t^2}{I_p G}l$,

V is the volume of the rod, l is the length of the rod, M_t is the torsion moment, I_p is the polar second moment of area, G is the shear modulus.

(48) How can the maximum stress be calculated in the case of pure torsion? Define the dangerous point of the cross section.

$$\left|\tau_{\varphi x}\right|_{\max} = \tau_{\max} = \frac{M_t}{I_p}\frac{D}{2} = \frac{M_t}{K_p} ,$$

D is the outer diameter of the cross section,
K_p is the polar section modulus of the cross section.

Dangerous point: point of the cross section where $\tau_{\varphi x \max}$ occurs.

(49) What are the free torsional warping and restrained torsional warping of a prismatic rod?

In the case of free torsional warping the displacement of the points of the rod along the centerline is not prevented.
In the case of restrained torsional warping points of the rod along the centerline cannot move.

(50) What are the loss of stability and the critical force?

Loss of stability: if a straight rod is deformed a little bit and it cannot regain its original shape.
Critical force: loss of stability can occur at this force.

(51) What are the equations of Euler's hyperbole and Tetmajer's straight line?

Euler's hyperbole: $\sigma_{crit} = \sigma_{crit}(\lambda) = \pi^2 \dfrac{E}{\lambda^2}$.

Tetmajer's straight line: $\sigma_{crit} = \sigma_{crit}(\lambda) = -\dfrac{R_{p0,2} - R_A}{\lambda_A}\lambda + R_{p0,2}$.

λ is the slenderness ratio of the rod, E is Young's modulus,
$R_{p0,2}$ is the yield stress of the material of the rod, R_A is the proportionality limit stress.

(52) How can stress state (of elementary environment) of point P of a solid body be interpreted?

Stress state (of elementary environment) of point P of a solid body is the set of all stress vectors $\vec{\rho}_n$ belonging to any plane (passing through point P) of normal unit vector $\vec{n}$.

(53) How can the stress state of point P be described unambiguously?

- The stress state of point P is described by stress tensor unambiguously:

$$\left[\underline{\underline{F}}_P\right] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}.$$

- The stress state of point P is described by three stress vectors unambiguously. The planes belonging to these vectors must be perpendicular to each other. (E.g. $\vec{\rho}_x, \vec{\rho}_y, \vec{\rho}_z$)

(54) What is the duality of the shear stresses τ ?

On any two perpendicular planes the magnitude of shear stresses τ perpendicular to the intersecting line of the planes are the same and they point alike to or away from the intersecting line. $\tau_{xy} = \tau_{yx}$, $\tau_{yz} = \tau_{zy}$, $\tau_{xz} = \tau_{zx}$.

(55) Give definition of principal axis, principal stress, and principal stress plane.

If on a plane perpendicular to unit vector $\vec{e}$ shear stress vector is zero $\vec{\tau}_e = \vec{0}$ (that is $\vec{\rho}_e = \sigma_e \vec{e}$), direction of $\vec{e}$ is principal axis, σ_e is principal stress and plane perpendicular $\vec{e}$ is principal stress plane.

(56) How can strain state (of elementary environment) of point P of a solid body be interpreted?

Strain state (of elementary environment) of point P of a solid body is the set of all changes of unit lengths in any directions of $\vec{n}$ (passing through point P) and changes of angles between perpendicular directions $\vec{n}$ and $\vec{m}$, $\vec{n} \cdot \vec{m} = 0$.

(57) How can the strain state of point P be described unambiguously?

- The strain state of point P is described by strain tensor unambiguously:

$$\left[\underline{\underline{A}}_P\right] = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_z & \frac{1}{2}\gamma_{zy} & \varepsilon_z \end{bmatrix}.$$

- The strain state of point P is described by displacements of the end points of three mutually perpendicular unit lengths unambiguously.

(58) Give definition of homogeneous, isotropic, linear elastic body.

Homogeneous: material properties are the same in all points of the body.

Isotropic: material properties do not depend on the direction.
Linear elastic: there is a linear relationship between stresses and strains.

(59) Write down the general Hooke's law and give the meaning of the quantities in it.

$$\underline{\underline{A}} = \frac{1}{2G}\left(\underline{\underline{F}} - \frac{\nu}{1+\nu} F_I \underline{\underline{E}}\right), \qquad \underline{\underline{F}} = 2G\left(\underline{\underline{A}} + \frac{\nu}{1-2\nu} A_I \underline{\underline{E}}\right)$$

$\underline{\underline{A}}$ - strain tensor, $\underline{\underline{F}}$ - stress tensor,
G - shear modulus, ν - Poisson's ratio
$F_I = \sigma_x + \sigma_y + \sigma_z = \sigma_1 + \sigma_2 + \sigma_3$ - first scalar invariant of the stress tensor,
$\sigma_1, \sigma_2, \sigma_3$ - principal stresses,
$A_I = \varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ - first scalar invariant of the strain tensor,
$\varepsilon_1, \varepsilon_2, \varepsilon_3$ - principal strains,
$\underline{\underline{E}}$ - unit tensor (matrix of the unit tensor: $\left[\underline{\underline{E}}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$).

(60) Give the definition of equivalent stress.

Equivalent stress characterizes the damaging effect of a stress state in a point unambiguously.

Equivalent stress reduces any arbitrary spatial stress state to a uniaxial stress state.

(61) When does a failure occur according to Coulomb theory? What is the definition of Coulomb equivalent stress? For what kind of materials does Coulomb theory describe the failure well?

- Failure occurs in a point if the maximum normal stress reaches the ultimate tensile/compressive strength.
- $\sigma_{equi}(Coulomb) = \max\left(|\sigma_1|, |\sigma_3|\right)$.
- Coulomb theory describes the failure well in the case of brittle materials if there is a dominant principal stress which is much larger than the other two principal stresses.

(62) When are two stress states identically dangerous in terms of failure according to Mohr theory? What is the definition of Mohr equivalent stress? For what kind of materials does Mohr theory describe the failure well?

- Two stress states are identically dangerous in terms of failure if the diameters of their maximal Mohr's circle are the same.
- $\sigma_{equi}(Mohr) = \sigma_1 - \sigma_3$.
- Mohr theory describes the failure well in the case of ductile materials.

(63) When are two stress states identically dangerous in terms of failure according to von Mises theory ([Maxwell-]Huber-Mises-Hencky theory)? What is the definition of von Mises equivalent stress? For what kind of materials does von Mises theory describe the failure well?

- Two stress states are identically dangerous in terms of failure if their strain energies u_T are the same.
- $\sigma_{equi}\left(HMH\right)=\sqrt{\frac{1}{2}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_1-\sigma_3\right)^2\right]}$, or

$$\sigma_{equi}\left(HMH\right)=\sqrt{\frac{1}{2}\left[\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+\left(\sigma_x-\sigma_z\right)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2\right)\right]}.$$

- von Mises theory describes the failure well in the case of ductile materials. The equivalent stress calculated from Mohr theory and the equivalent stress derived from von Mises theory differ only a little bit.

(64) Describe the general idea of sizing to maximum stress and checking of rod/beam structures.

- Looking for the dangerous cross section(s) of the structure. That cross section is dangerous on which the internal forces have maximum.
- Looking for the dangerous points on the dangerous cross section. That point is dangerous in which σ_{equi} has maximum.
- Sizing in the dangerous point(s), checking.
  $\sigma_{equi\max}\le\sigma_{allowed}$.

(65) What are the combined internal forces? Describe the superposition principle of rod structures.

- Combined internal forces: if there are at least two non-zero coordinates of the internal forces in the rod.
  For example: N, $M_b \neq 0$, or M_b, $M_t \neq 0$, etc.
- In the case of combined internal forces the states of simple internal forces are summed.

(66) Define the zero line. What is the equation of the zero line in the case of tension/compression and symmetric bending?

One can talk of zero line only in the case of uniaxial stress state. The zero line is formed by points of the cross section where σ_x is zero.
The equation of the zero line in the case of tension/compression and symmetric bending:

$$\sigma_x=0=\frac{N}{A}+\frac{M_{hz}}{I_z}y_0 \quad \Rightarrow \quad y_0=-\frac{N}{M_{hz}}\frac{I_z}{A}.$$

(67) Write down both of interpretations of unsymmetric bending.

If bending moment vector $\vec{M}_b$ is not parallel to any principal axes of the cross section.

If bending moment vector $\vec{M}_b$ is not parallel to the zero line.

(68) How can stress σ_x and equation of the zero line be determined in the case of unsymmetric bending?

Stress in the rod: $\sigma_x = \frac{M_{bz}}{I_z} y + \frac{M_{by}}{I_y} z$ where z and y are principal axes of the cross section.

The equation of the zero line: $\sigma_x = 0 = \frac{M_{bz}}{I_z} y + \frac{M_{by}}{I_y} z \quad \Rightarrow \quad y = y(z) = -\frac{M_{by}}{M_{bz}} \frac{I_z}{I_y} z$.

(69) What is an eccentric tension (compression)?

If the resultant force of the force system acting on the cross section is parallel to the axis of the rod and it does not pass through the center of mass of the cross section.

(70) How can stress σ_x and dangerous points of the cross section be determined in the case of eccentric tension (compression)?

- Stress in the rod: $\sigma_x = \sigma_{x_N} + \sigma_{x_B} = \frac{N}{A} + \frac{M_{bz}}{I_z} y + \frac{M_{by}}{I_y} z$ where z and y are principal axes of the cross section.
- Dangerous points of the cross section are the points most distant from the zero line in the case of eccentric tension (compression).

(71) Give definition of kernel/core in the case of of eccentric tension (compression).

Kernel/core is set of points of application on which force F generates stresses with only one kind of sign.

(72) Under what assumptions is the approximation of bending and shearing of a rod good?

- There occur only symmetric bending and axes z and y are principal axes about center of mass of the cross section.
- Stresses $\vec{\tau}_x$ originating from shearing on a line parallel to axis z intersect each other in a common point on axis y.
- τ_{xy} is constant on a line parallel to axis z.

(73) How can stresses of the rod be determined in the case of bending and shearing?

From bending: $\sigma_x = \frac{M_{bz}}{I_z} y$, from shearing: $\tau_{yx} = -\frac{T_y S_{1z}(y)}{I_z a(y)}$ where

T_y - shear force, M_{bz} - bending moment,

I_z - second moment of area of the cross section with respect to axis z,

$S_{1z}(y)$ - static moment of part of the cross section above line $y = \text{const.}$ with respect to axis z,

$a(y)$ - length of common part of line $y = \text{const.}$ and the cross section.

(74) Define shear center of the cross section and describe its role in determining the internal forces in the cross section.

Shear center is the point of application of the resultant force of the shear stresses of the cross section.

If the shear force resulting from the load does not pass through the shear center C_T of the cross section, the cross section is not only bent and sheared but also twisted. Torsion moment is the moment of the resultant force of the load about the shear center.

(75) How can the strain energy of rod structures be calculated?

$$U=\int\limits_{(V)} u\,dV = \underbrace{U_N}_{\substack{\text{strain energy}\\ \text{of tension or compression}}} + \underbrace{U_B}_{\substack{\text{strain energy}\\ \text{of bending}}} + \underbrace{U_T}_{\substack{\text{strain energy}\\ \text{of torsion}}} + \underbrace{U_S}_{\substack{\text{strain energy}\\ \text{of shearing}}}$$

If $U_S \approx 0$, then $U=\dfrac{1}{2}\displaystyle\int\limits_{(l)}\left(\frac{N^2}{AE}+\frac{M_{bz}^2}{I_zE}+\frac{M_{by}^2}{I_yE}+\frac{M_t^2}{I_pG}\right)dx$ where

E - Young's modulus, G - shear modulus,
A - area of the cross section,
I_z, I_y - planar second moments of area with respect to axis z and y, I_p - polar second moment of area,
N - normal force, M_{bz} , M_{by} - bending moment, M_t - torsion moment.

(76) Describe the most commonly used form of Betti's theorem.

$W_{21}=U_{12}$.

Work of force system 2 on strains caused by force system 1 equals to the "mixed" part of the strain energy.

$W_{21}=\sum\limits_{i=1}^{n}\vec{F}_i''\cdot\vec{t}_i'+\sum\limits_{j=1}^{m}\vec{M}_j''\cdot\vec{\varphi}_j'$ is the work of force system 2 on strains caused by force system 1.

$U_{12}=\displaystyle\int\limits_{(l)}\left(\frac{N'N''}{AE}+\frac{M_{bz}'M_{bz}''}{EI_z}+\frac{M_{by}'M_{by}''}{EI_y}+\frac{M_t'M_t''}{I_pG}\right)dx$ is the "mixed" part of the strain energy.

(77) Describe planar form of Castigliano's theorem.

$$u_i=\frac{\partial U}{\partial F_{xi}},\quad v_i=\frac{\partial U}{\partial F_{yi}},\quad \varphi_i=\frac{\partial U}{\partial M_{zi}}.$$

First and second relation: displacement of the point of application of the load F_i in the direction of force F_i equals to the derivative of the strain energy of the structure with respect to F_i.

Third relation: rotation φ_i of the cross section in the point of application of the load M_{z_i} equals to the derivative of the strain energy of the structure with respect to M_{z_i}.

(78) Define the statically determinate and undeterminate structures.

Statically determinate structure:

If the number of coordinates of the unknown support and inner forces equals to the number of the scalar equations which describe the equilibrium of the structure and the system of equations can be solved unambiguously.

Statically undeterminate structure:
If the number of coordinates of the unknown support and inner forces is greater than the number of the scalar equations which describe the equilibrium of the structure.

(79) What are the steps of the determination of the support force system of a statically undeterminate structure?

- Making the structure statically determinate by omitting constraint(s).
- Prescribing such kinematic restrictions which replace the omitted constraints.
- Applying of Castigliano's theorem for determination of coordinates of support forces/moments at the kinematic restrictions.
- Determining the other coordinates of support forces/moments from the equilibrium equations of the statics.