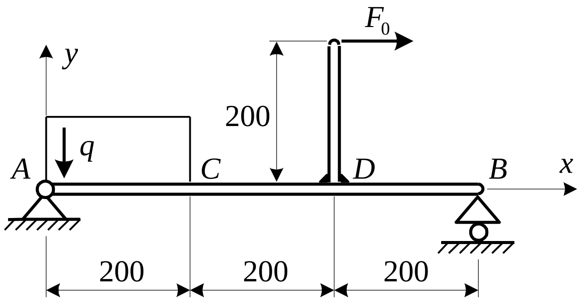
9. MECHANIKA-STATIKA GYAKORLAT
(kidolgozta: Triesz Péter, egy. ts.; Tarnai Gábor, mérnöktanár)

Rúdszerkezetek igénybevételei

9.1. Példa

Adott:

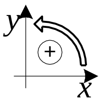


$q = 20\,\text{kN/m} \quad \rightarrow \quad F_q = 4\,\text{kN}$,
$F_0 = 5\,\text{kN}$.

Feladat:

a. Határozza meg a támasztóerőket!
b. Számítsa ki az igénybevételeket az *A*, *B*, *C* és *D* keresztmetszetekben!

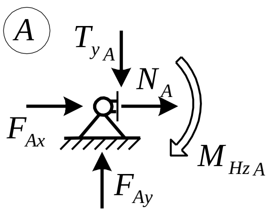
Megoldás:



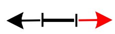
a. A támasztó erőrendszer kiszámítása:

$$F_x = 0 = F_{Ax} + F_0 \quad \Rightarrow \quad F_{Ax} = -F_0 = -5\,\text{kN}\,(\leftarrow),$$
$$F_y = 0 = F_{Ay} - F_q + F_{By} \quad \Rightarrow \quad F_{Ay} = 1{,}6\dot{6}\,\text{kN}\,(\uparrow),$$
$$M_a = 0 = -0{,}1F_q - 0{,}2F_0 + 0{,}6F_{By} \quad \Rightarrow \quad F_{By} = 2{,}3\dot{3}\,\text{kN}\,(\uparrow).$$

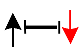
b. Az igénybevételek számításakor *a rúd meghagyott részének egyensúlyát* vizsgáljuk.



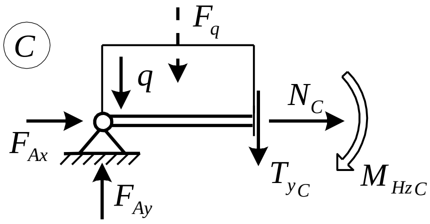
$$F_x = 0 = F_{Ax} + N_A \quad \Rightarrow$$
$$N_A = -F_{Ax} = -(-5) = 5\,\text{kN} \rightarrow$$



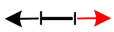
$$F_y = 0 = F_{Ay} - T_A$$
$$\Rightarrow \quad T_A = F_{Ay} = 1{,}6\dot{6}\,\text{kN} \downarrow$$



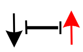
$$M_a = 0 = M_{Hz}(A) = 0$$



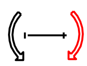
$$F_x = 0 = F_{Ax} + N_C \quad \Rightarrow$$
$$N_C = -F_{Ax} = -(-5) = 5\,\text{kN} \rightarrow$$

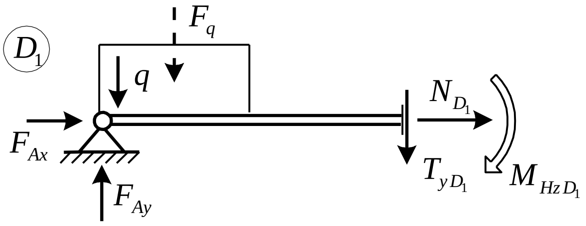


$$F_y = 0 = F_{Ay} - F_q - T_C$$
$$\Rightarrow \quad T_C = F_{Ay} - F_q = -2{,}3\dot{3}\,\text{kN} \uparrow$$



$$M_c = 0 = -0{,}2 \cdot F_{Ay} + 0{,}1 \cdot F_q - M_{HzC}$$
$$\Rightarrow \quad M_{Hz}(C) = 0{,}06\dot{6}\,\text{kNm}$$





$F_x = 0 = F_{Ax} + N_{D_1} \Rightarrow$

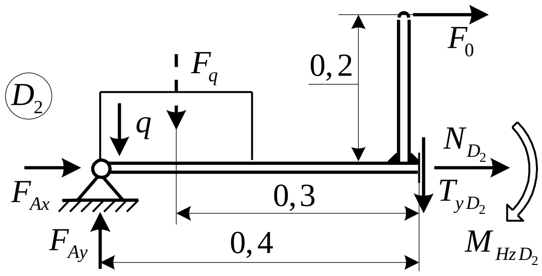
$N_{D_1} = -F_{Ax} = 5 \text{ kN} \rightarrow$

$F_y = 0 = F_{Ay} - F_q - T_{D_1}$

$\Rightarrow T_{D_1} = F_{Ay} - F_q = -2,3\dot{3} \text{ kN} \uparrow$

$M_{D_1} = 0 = -0,4 \cdot F_{Ay} + 0,3 \cdot F_q - M_{HzD_1}$

$\Rightarrow M_{Hz}(D_1) = 0,53\dot{3} \text{ kNm}$



$F_x = 0 = F_{Ax} + F_0 + N_{D_2} \Rightarrow$

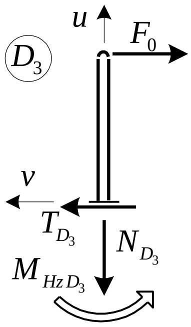
$N_{D_2} = -F_{Ax} - F_0 = -(-5) - 5 = 0 \text{ kN}$

$F_y = 0 = F_{Ay} - F_q - T_{D_2}$

$\Rightarrow T_{D_2} = -2,3\dot{3} \text{ kN} \uparrow$

$M_{D_2} = 0 = -0,4 \cdot F_{Ay} + 0,3 \cdot F_q - 0,2 \cdot F_0 - M_{HzD_2}$

$\Rightarrow M_{Hz}(D_2) = -0,46\dot{6} \text{ kNm}$

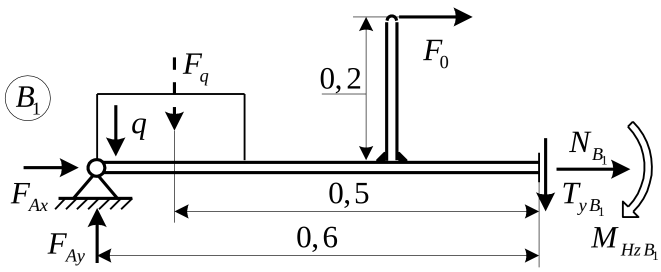


$F_u = 0 = -N_{D_3} \Rightarrow N_{D_3} = 0$

$F_v = 0 = T_{D_3} - F_0 \Rightarrow T_{D_3} = 5 \text{ kN} \leftarrow$

$M_{D_3} = 0 = M_{HzD_3} - 0,2F_0$

$\Rightarrow M_{Hz}(D_3) = 1 \text{ kNm}$



$F_x = 0 = F_{Ax} + F_0 + N_{B_1} \Rightarrow$

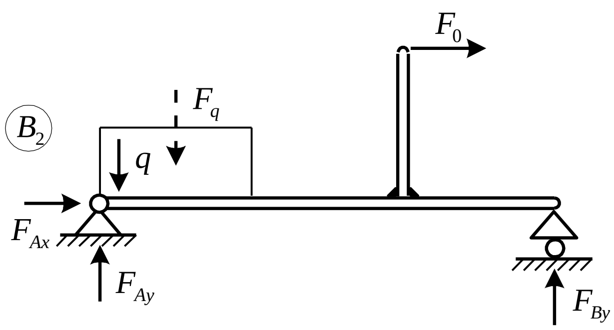
$N_{B_1} = -F_{Ax} - F_0 = -(-5) - 5 = 0 \text{ kN}$

$F_y = 0 = F_{Ay} - F_q - T_{yB_1}$

$\Rightarrow T_{B_1} = -2,3\dot{3} \text{ kN} \uparrow$

$M_{B_1} = 0 = -0,6F_{Ay} + 0,5F_q - 0,2F_0 - M_{HzB_1}$

$\Rightarrow M_{Hz}(B_1) = 0$

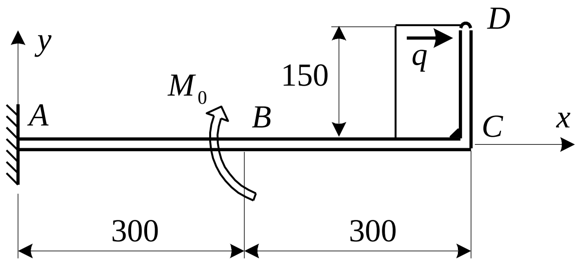


$N_{B_2} = 0$

$T_{B_2} = 0$

$M_{Hz}(B_2) = 0$

9.2. Példa



Adott:

$q = 30\text{ kN/m} \quad \rightarrow \quad F_q = 4{,}5\text{ kN}$,

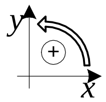
$M_0 = 10\text{ kNm}$.

Feladat:

a. Határozza meg a támasztóerőket!

b. Számítsa ki az igénybevételeket az *A*, *B*⁻, *B*⁺, *C*⁻ és *C*⁺ keresztmetszetekben!

Megoldás:



a. A támasztó erőrendszer kiszámítása:

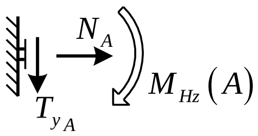
$$F_x = 0 = F_{Ax} + F_q \quad \Rightarrow \quad F_{Ax} = -F_q = -4{,}5\text{ kN} (\leftarrow),$$

$$F_y = 0 = F_{Ay},$$

$$M_a = 0 = M_A - M_0 - 0{,}075 \cdot F_q \quad \Rightarrow \quad M_A = 10{,}3375\text{ kNm}(\circlearrowleft)$$

b. Az igénybevételeket ezúttal úgy határozzuk meg, hogy *a rúd elhagyott részén ébredő erőrendszert redukáljuk* a megmaradt rész végének súlypontjába.

(A)



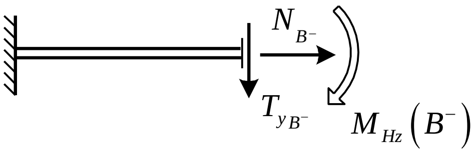
$N_A = F_{ex}(A) = F_q = 4{,}5\text{ kN} \rightarrow$

$T_{yA} = F_{ey}(A) = 0$

$M_A = -M_0 - 0{,}075 \cdot F_q = -10{,}3375\text{ kNm}$

$M_{Hz}(A) = 10{,}3375\text{ kNm}$

(B⁻)



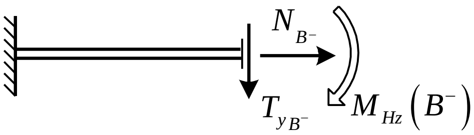
$N_{B^-} = F_q = 4{,}5\text{ kN} \rightarrow$

$T_{yB^-} = 0$

$M_B = -M_0 - 0{,}075 \cdot F_q = -10{,}3375\text{ kNm}$

$M_{Hz}(B^-) = 10{,}3375\text{ kNm}$

(B⁺)



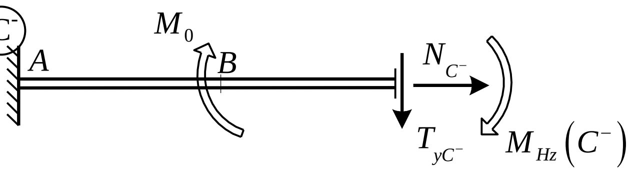
$N_{B^+} = F_q = 4{,}5\text{ kN} \rightarrow$

$T_{yB^+} = 0$

$M_{B^+} = -0{,}075 \cdot F_q = -0{,}3375\text{ kNm}$

$M_{Hz}(B^+) = 0{,}3375\text{ kNm}$

(C⁻)



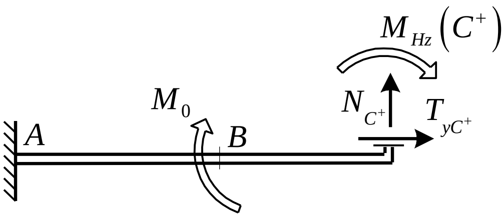
$N_{C^-} = F_q = 4{,}5\text{ kN} \rightarrow$

$T_{yC^-} = 0$

$M_{C^-} = -0{,}075 \cdot F_q = -0{,}3375\text{ kNm}$

$M_{Hz}(B^+) = 0{,}3375\text{ kNm}$

(C⁺)



$N_{C^+} = F_{ey} = 0$

$T_{yC^+} = F_{ex} = F_q = 4{,}5\text{ kN} \rightarrow$

$M_{C^+} = -0{,}075 \cdot F_q = -0{,}3375\text{ kNm}$

$M_{Hz}(C^+) = 0{,}3375\text{ kNm}$

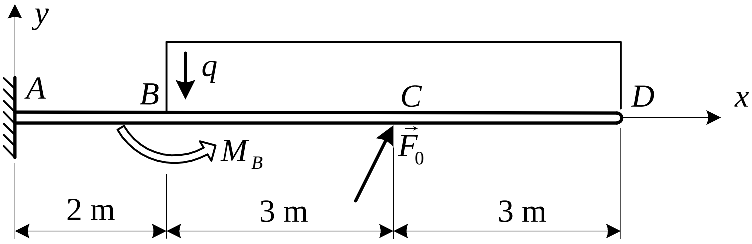
9.3. Példa

Adott:

$\vec{F}_0 = \left(5\vec{i} + 10\vec{j}\right)\text{kN}$,

$q = 2\ \text{kN/m} \quad \rightarrow \quad F_q = 12\ \text{kN}$,

$M_B = 8\ \text{kNm}$.

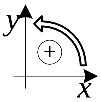


Feladat:

a. Határozza meg a támasztóerőket!

b. Számítsa ki az igénybevételeket az A, B^-, B^+, C^-, C^+ és D keresztmetszetekben!

Megoldás:



a. A támasztó erőrendszer kiszámítása:

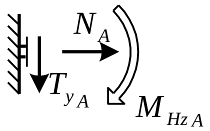
$$F_x = 0 = F_{Ax} + F_{0x} \quad \Rightarrow \quad F_{Ax} = -5\ \text{kN}\ (\leftarrow)$$

$$F_y = 0 = F_{Ay} - F_q + F_{0y} \quad \Rightarrow \quad F_{Ay} = F_q - F_{0y} = 12 - 10 = 2\ \text{kN}\ (\uparrow)$$

$$M_a = 0 = M_A + M_0 - 5 \cdot F_q + 5 \cdot F_{0y} \quad \Rightarrow \quad M_A = 2\ \text{kNm}\ (\circlearrowleft)$$

b. Az igénybevételeket határozzuk meg úgy, hogy <u>*a rúd elhagyott részén ébredő erőrendszert redukáljuk*</u> a megmaradt rész végének súlypontjába

Ⓐ



$$F_{ex}(A) = N_A = F_{0x} = 5\ \text{kN} \rightarrow$$

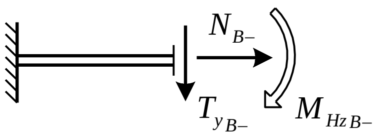
$$F_{ye}(A) = -F_q + F_{0y} = -2\ \text{kN} \downarrow \quad \Rightarrow$$

$$T_{yA} = 2\ \text{kN}$$

$$M_A = M_B - 5 \cdot F_q + 5 \cdot F_{0y} = -2\ \text{kNm}$$

$$M_{HzA} = 2\ \text{kNm}$$

B^-



$$F_{ex}(B^-) = N_{B^-} = F_{0x} = 5\ \text{kN} \rightarrow$$

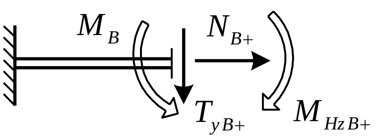
$$F_{ey}(B^-) = -F_q + F_{0y} = -2\ \text{kN} \downarrow$$

$$T_{yB^-} = 2\ \text{kN}$$

$$M_{B^-} = M_B - 3 \cdot F_q + 3 \cdot F_{0y} = 2\ \text{kNm}$$

$$M_{HzB^-} = -2\ \text{kNm}$$

B^+



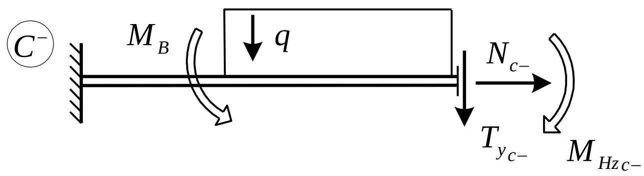
$$F_{ex}(B^+) = N_{B^+} = F_{0x} = 5\ \text{kN} \rightarrow$$

$$F_{ey}(B^+) = -F_q + F_{0y} = -2\ \text{kN} \downarrow$$

$$T_{yB^+} = 2\ \text{kN}$$

$$M_{B^+} = -3 \cdot F_q + 3 \cdot F_{0y} = -6\ \text{kNm}$$

$$M_{HzB^+} = 6\ \text{kNm}$$



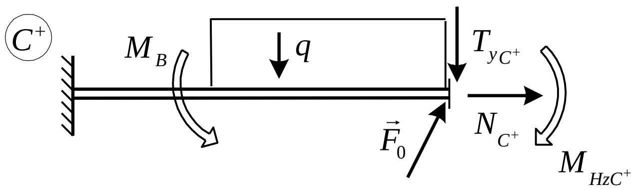
$$F_{exC^-} = N_{C^-} = F_{0x} = 5\text{ kN} \rightarrow$$

$$F_{eyC^-} = -\frac{F_q}{2} + F_{0y} = 4\text{ kN} \uparrow$$

$$T_{yC^-} = -4\text{ kN}$$

$$M_{C^-} = -1{,}5 \cdot \frac{F_q}{2} = -9\text{ kNm}$$

$$M_{HzC^-} = 9\text{ kNm}$$



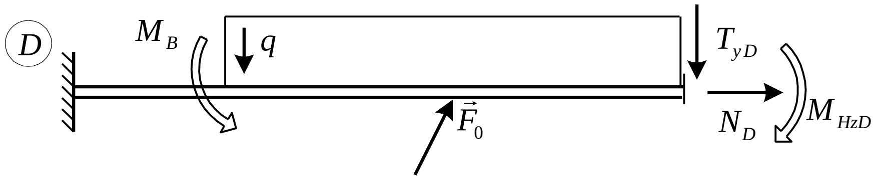
$$F_{exC^+} = N_{C^+} = 0\text{ kN}$$

$$F_{eyC^+} = -\frac{F_q}{2} = -6\text{ kN} \downarrow$$

$$T_{yC^+} = 6\text{ kN}$$

$$M_{C^+} = -1{,}5 \cdot \frac{F_q}{2} = -9\text{ kNm}$$

$$M_{HzC^+} = 9\text{ kNm}$$



$$N_D = 0\text{ kN}$$

$$T_D = 0\text{ kN}$$

$$M_{HzD} = 0\text{ kNm}$$

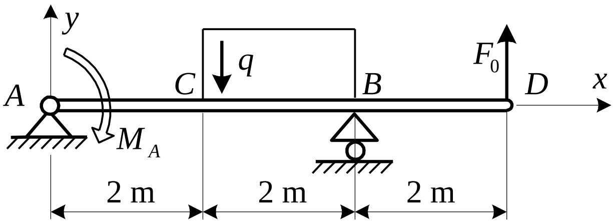
9.4. Példa

Adott:

$q = 3\,\text{kN/m} \quad \rightarrow \quad F_q = 6\,\text{kN}$,

$F_0 = 4\,\text{kN}$,

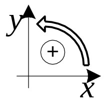
$M_A = 5\,\text{kNm}$.



Feladat:

a. Határozza meg a támasztóerőket!
b. feladat rész kiírása inkább: Számítsa ki az igénybevételeket az *A*, *B*⁻, *B*⁺, *C*⁻, és D^-, D^+ keresztmetszetekben!

Megoldás:



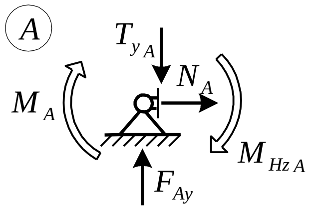
a. A támasztó erőrendszer kiszámítása:

$F_x = 0 = F_{Ax}$,

$$F_y = 0 = F_{Ay} - F_q + F_{By} + F_0 \Rightarrow F_{Ay} = \frac{9}{4}\,\text{kN}\,(\uparrow),$$

$$M_a = 0 = -M_A - 3\cdot F_q + 4\cdot F_{By} + 6\cdot F_0 \Rightarrow F_{By} = \frac{M_A + 3F_q - 6F_0}{4} = -\frac{1}{4}\,\text{kN}\,(\downarrow).$$

b. Most az igénybevételek számításakor *a rúd meghagyott részének egyensúlyát* vizsgáljuk.



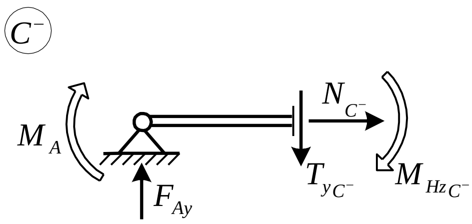
$F_x = 0 = N_A \quad \Rightarrow \quad N_A = 0$

$F_y = 0 = F_{Ay} - T_A$

$\Rightarrow \quad T_A = F_{Ay} = 2,25\,\text{kN}\downarrow$

$M_a = 0 = -M_A - M_{HzA}$

$\Rightarrow M_{HzA} = -M_A = -5\,\text{kN}$



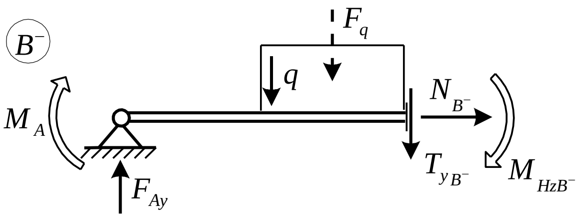
$F_x = 0 = N_{C^-} \quad \Rightarrow \quad N_{C^-} = 0$

$F_y = 0 = F_{Ay} - T_{yC^-} \Rightarrow$

$T_{yC^-} = F_{Ayq} = 2,25\,\text{kN}\downarrow$

$M_{c^-} = 0 = -M_A - 2F_{Ay} - M_{HzC^-} \quad \Rightarrow$

$M_{HzC^-} = -M_A - 2F_{Ay} = -9,5\,\text{kN}$



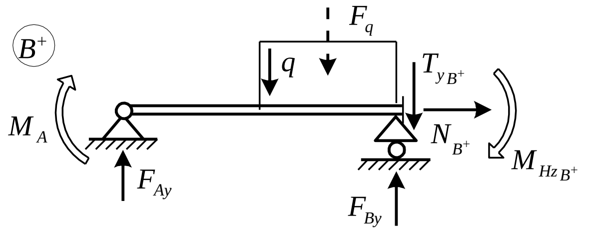
$F_x = 0 = N_{B^-} \quad \Rightarrow \quad N_{B^-} = 0$

$F_y = 0 = F_{Ay} - F_q - T_{B^-} \quad \Rightarrow$

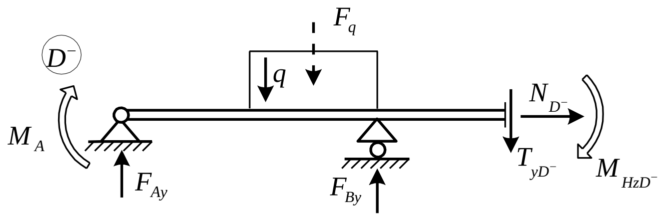
$T_{B^-} = F_{Ay} - F_q = -3,75\,\text{kN}\uparrow$

$M_{b^-} = 0 = -M_A - 4\cdot F_{Ay} + 1\cdot F_q - M_{HzB^-} \quad \Rightarrow$

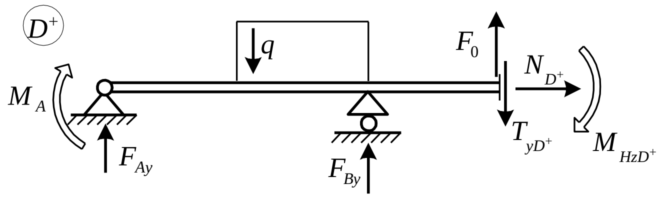
$M_{HzB^-} = -M_A - 4F_{Ay} + F_q = -8\,\text{kN}$



$$F_x = 0 = N_{B^+} \quad \Rightarrow \quad N_{B^+} = 0$$
$$F_y = 0 = F_{Ay} - F_q + F_{By} - T_{yB^+} \quad \Rightarrow$$
$$T_{yB^+} = \frac{9}{4} - 6 + \left(-\frac{1}{4}\right) = -4 \text{ kN} \uparrow$$
$$M_{b^+} = 0 = -M_A - 4 \cdot F_{Ay} + 1 \cdot F_q - M_{HzB^+}$$
$$\Rightarrow \quad M_{HzB^+} = -8 \text{ kNm} \curvearrowleft$$



$$N_{D^-} = 0$$
$$F_y = 0 = F_{Ay} - F_q + F_{By} - T_{yD^-}$$
$$\Rightarrow \quad T_{yD^-} = -4 \text{ kN} \uparrow$$
$$M_{b^-} = 0 = -M_A - 6F_{Ay} + 3F_q + 2F_{By} - M_{HzD^-}$$
$$\Rightarrow \quad M_{HzD^-} = 0 \text{ kNm}$$



$$N_{D^+} = 0$$
$$F_y = 0 = F_{Ay} - F_q + F_{By} + F_0 - T_{yD^+}$$
$$\Rightarrow \quad T_{yD^+} = 0 \text{ kN}$$
$$M_{d^+} = 0 = -M_A - 6F_{Ay} + 3F_q + 2F_{By} - M_{HzD^+}$$
$$\Rightarrow \quad M_{HzD^+} = 0 \text{ kNm}$$