SZÉCHENYI ISTVÁN EGYETEM

ALKALMAZOTT MECHANIKA TANSZÉK

7. MECHANIZMUSOK GYAKORLAT

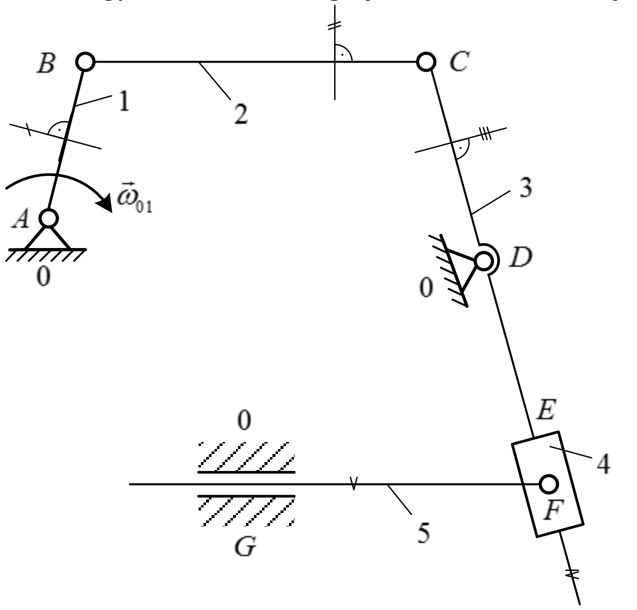
(kidolgozta: Bojtár Gergely egy. Ts; Tarnai Gábor mérnöktanár.)

Mechanizmusok gyorsulásállapota

7.1.

Adott: A mechanizmus méretei, pillanatnyi helyzete, és a meghajtás: $\vec{\omega}_{01}$=állandó.

Feladat: gyorsulásábra megrajzolása az adott helyzetben.



Megoldás:

$$\text{Szerkezeti képlet: } \overset{\downarrow}{A}BCD \leftarrow EFG$$

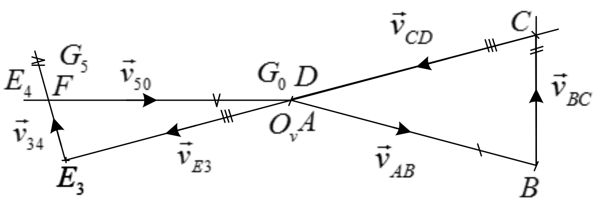
$$h_g = (1+1+1+1-3)+(1+1+1-3) = 1+0 = 1$$

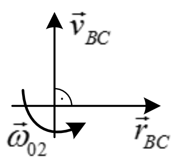
$$h_k = (1-1)+(0-0) = 0+0 = 0 \quad \Rightarrow \quad \text{egyszerű mechanizmus}$$

Sebességállapot:

$$\underline{\vec{v}}_{AB} + \underline{\vec{v}}_{BC} + \underline{\vec{v}}_{CD} = \vec{0} \qquad \text{1. lánc}$$

$$\vec{v}_{E_3} + \vec{v}_{34} + \vec{v}_{50} = \vec{0} \qquad \text{2. lánc, ahol} \qquad \vec{v}_{E_3} = \vec{v}_{DE}$$





$$\vec{v}_{BC} = \vec{v}_C - \vec{v}_B = \vec{\omega}_{02} \times \vec{r}_{BC} \quad \Rightarrow \quad \omega_{02} = \frac{v_{BC}}{r_{BC}},$$

$$\vec{v}_{CD} = \vec{v}_D - \vec{v}_C = \vec{\omega}_{03} \times \vec{r}_{CD} \quad \Rightarrow \quad \omega_{03} = \frac{v_{CD}}{r_{CD}},$$

<u>Gyorsulási állapot</u>:

1. lánc: $\vec{a}_{AB} + \vec{a}_{BC} + \vec{a}_{CD} = \vec{0}$ $\qquad {}_0A_1B_2C_3D_0$ zárt lánc

Egy síkbeli, merev testen – a 2. tagon – lévő két pont gyorsulása közötti összefüggésből:

$$\vec{a}_C = \vec{a}_B + \vec{\varepsilon}_{02} \times \vec{r}_{BC} - \omega_{02}^2 \vec{r}_{BC} \quad \Rightarrow$$

$$\vec{a}_{BC} = \vec{a}_C - \vec{a}_B = \underbrace{\vec{\varepsilon}_{02} \times \vec{r}_{BC}}_{\vec{b}_{BC}} \underbrace{-\omega_{02}^2 \vec{r}_{BC}}_{\vec{c}_{BC}} = \vec{b}_{BC} + \vec{c}_{BC} \quad \Rightarrow \quad \vec{b}_{BC} \perp \vec{r}_{BC}, \quad \vec{c}_{BC} \parallel \vec{r}_{BC}.$$

A teljes lánc: $\left(\vec{b}_{AB} + \vec{c}_{AB}\right) + \left(\vec{b}_{BC} + \vec{c}_{BC}\right) + \left(\vec{b}_{CD} + \vec{c}_{CD}\right) = \vec{0}$

$$\vec{b}_{AB} = \underbrace{\vec{\varepsilon}_{01}}_{\vec{0}} \times \vec{r}_{AB} = \vec{0} \quad \leftarrow \quad \vec{\omega}_{01} = áll.$$

$$\vec{c}_{AB} = -\omega_{01}^2 \vec{r}_{AB} \qquad \rightarrow \quad \vec{c}_{AB} \parallel \vec{r}_{AB} \qquad (\omega_{01} \text{ adott} \quad \Rightarrow \quad \underline{\underline{\vec{c}}}_{AB} \text{ ismert})$$

$$\vec{b}_{BC} = \vec{\varepsilon}_{02} \times \vec{r}_{BC} \qquad \rightarrow \quad \vec{b}_{BC} \perp \vec{r}_{BC}$$

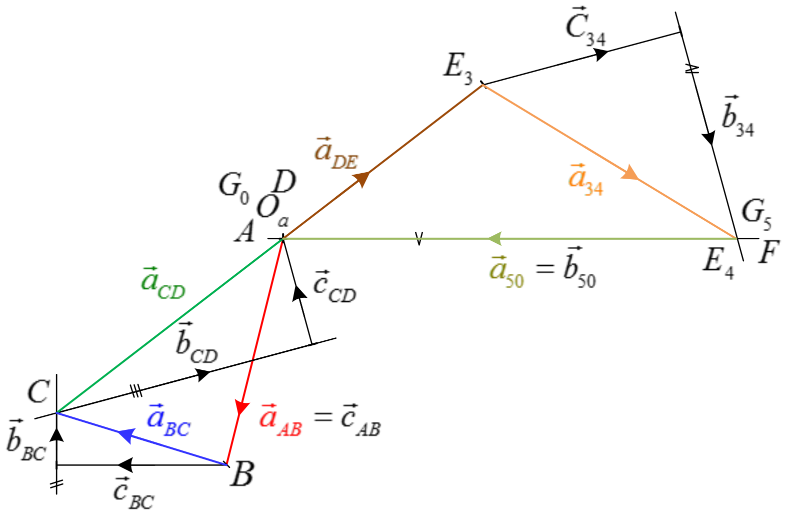
$$\vec{c}_{BC} = -\omega_{02}^2 \vec{r}_{BC} \qquad \rightarrow \quad \vec{c}_{BC} \parallel \vec{r}_{BC} \qquad \left(\omega_{02} = \frac{v_{BC}}{r_{BC}} \quad \Rightarrow \quad \underline{\underline{\vec{c}}}_{BC} \text{ ismert}\right)$$

$$\vec{b}_{CD} = \vec{\varepsilon}_{03} \times \vec{r}_{CD} \qquad \rightarrow \quad \vec{b}_{CD} \perp \vec{r}_{CD}$$

$$\vec{c}_{CD} = -\omega_{03}^2 \vec{r}_{CD} \qquad \rightarrow \quad \vec{c}_{CD} \parallel \vec{r}_{CD} \qquad \left(\omega_{03} = \frac{v_{CD}}{r_{CD}} \quad \Rightarrow \quad \underline{\underline{\vec{c}}}_{CD} \text{ ismert}\right)$$

$$\underbrace{\underline{\underline{\vec{c}}}_{AB}}_{\vec{a}_{AB}} + \underbrace{\underline{\underline{\vec{c}}}_{BC} + \underline{\vec{b}}_{BC}}_{\vec{a}_{BC}} + \underbrace{\underline{\vec{b}}_{CD} + \underline{\underline{\vec{c}}}_{CD}}_{\vec{a}_{CD}} = \vec{0}, \qquad \left.\begin{array}{l} \vec{b}_{BC} \\ \vec{b}_{CD} \end{array}\right\} \Rightarrow \quad C \text{ pont.}$$

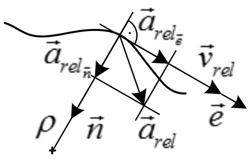
(A gyorsulásvektorok sorrendje nem, csak a zárójelben lévő tagok cserélhetők föl.)



$$\vec{a}_{AB} + \vec{a}_{BC} + \vec{a}_{CD} = \vec{0}$$
$$\vec{a}_{DE} + \vec{a}_{34} + \vec{a}_{50} = \vec{0}$$

2. lánc: $\vec{a}_{DE} + \vec{a}_{34} + \vec{a}_{50} = \vec{0}$ $\quad {}_0D_3E_4F_5G_0$ zárt lánc

Két különböző merev test – a 3. és 4. tag – esetében a relatív mozgást vizsgáljuk:
$\vec{a}_{absz} = \vec{a}_{száll} + \vec{a}_{rel} + \vec{a}_{cor} \quad \Rightarrow \quad \vec{a}_{E_4} = \vec{a}_{E_3} + \vec{a}_{rel} + \vec{a}_{cor}$,



$\vec{a}_{E_4}; \vec{a}_{E_3}$ abszolút gyorsulások.

$\vec{a}_{E_4} - \vec{a}_{E_3} = \vec{a}_{34} = \vec{a}_{rel} + \vec{a}_{cor} = \left(\vec{a}_{rel_{\vec{e}}} + \vec{a}_{rel_{\vec{n}}}\right) + \vec{a}_{cor}$

$\vec{e}$: pályagörbe érintő, $\vec{n}$: pályagörbe sugár-középpont irányú,

$\vec{a}_{rel_{\vec{e}}} = \vec{b}_{34} \quad \to \vec{b}_{34} \parallel \vec{v}_{34}$: 4. tagban csúszka irányú,

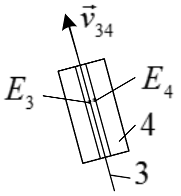
$\vec{v}_{34}$ egyenes vonalú mozgást végez $\to \rho_3 = \infty$:pályagörbe sugara, $\quad \Rightarrow \quad \vec{c}_{34} = \vec{0}$.

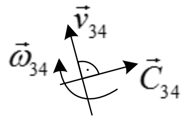
$\vec{a}_{cor} = \vec{C}_{34} = 2\vec{\omega}_{03} \times \vec{v}_{34} \quad \Rightarrow \quad C_{34} = 2\omega_{03} \cdot v_{34} \quad \Rightarrow \vec{C}_{34} \perp \vec{v}_{34}$;

Teljes lánc: $\vec{a}_{DE} + \vec{a}_{34} + \vec{a}_{50} = \vec{0}$

$$\left(\underline{\underline{\vec{b}}}_{DE} + \underline{\underline{\vec{c}}}_{DE}\right) + \left(\underline{\vec{b}}_{34} + \underline{\underline{\vec{c}}}_{34} + \underline{\underline{\vec{C}}}_{34}\right) + \left(\underline{\vec{b}}_{50} + \underline{\underline{\vec{c}}}_{50} + \underline{\underline{\vec{C}}}_{50}\right) = \vec{0}$$

$$\underline{\underline{\vec{b}}}_{DE} + \underline{\underline{\vec{c}}}_{DE} = \underline{\underline{\vec{a}}}_{DE}; \quad \varepsilon_{03} = \frac{a_{DE}}{r_{DE}} = \frac{a_{CD}}{r_{CD}} \quad \to a_{DE} = a_{CD}\frac{r_{DE}}{r_{CD}} \quad \Rightarrow \quad \underline{\underline{\vec{a}}}_{DE} \parallel \vec{a}_{CD} \text{ ismert,}$$





$\vec{b}_{34} \parallel \vec{v}_{34}$ csúszka irányú (IV); $\quad \vec{c}_{34} = \vec{0} \leftarrow \rho_3 = \infty$; $\quad C_{34} = 2\omega_{03} \cdot v_{34} \quad \Rightarrow \vec{C}_{34} \perp \vec{v}_{34}$;

$\vec{b}_{50}$: G csúszka irányú (V); $\quad c_{50} = \dfrac{v_{50}^2}{\rho_5} = 0 \quad \leftarrow \rho_5 = \infty$; $\quad \vec{C}_{50} = 2\underbrace{\vec{\omega}_{05}}_{=\vec{0}} \times \vec{v}_{50} = \vec{0}$;

$$\left(\underline{\underline{\vec{b}}}_{DE} + \underline{\underline{\vec{c}}}_{DE}\right) + \left(\underline{\vec{b}}_{34} + \underbrace{\underline{\underline{\vec{c}}}_{34}}_{\vec{0}} + \underline{\underline{\vec{C}}}_{34}\right) + \left(\underline{\vec{b}}_{50} + \underbrace{\underline{\underline{\vec{c}}}_{50}}_{\vec{0}} + \underbrace{\underline{\underline{\vec{C}}}_{50}}_{\vec{0}}\right) = \vec{0},$$

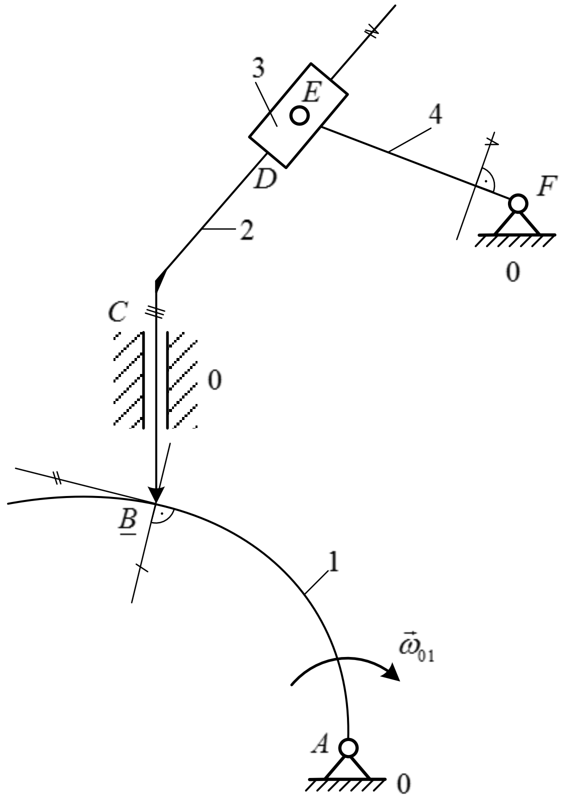
(A gyorsulásvektorok sorrendje nem, csak a zárójelben lévő tagok cserélhetők föl.)

$$\underbrace{\underline{\underline{\vec{b}}}_{DE} + \underline{\underline{\vec{c}}}_{DE}}_{\vec{a}_{DE}} + \underbrace{\underline{\underline{\vec{C}}}_{34} + \underline{\vec{b}}_{34}}_{\vec{a}_{34}} + \underbrace{\underline{\vec{b}}_{50}}_{\vec{a}_{50}} = \vec{0} \quad \Rightarrow \quad \left.\begin{matrix}\vec{b}_{34}\\ \vec{b}_{50}\end{matrix}\right\} \quad \Rightarrow E_4, F, G_5 \,.$$

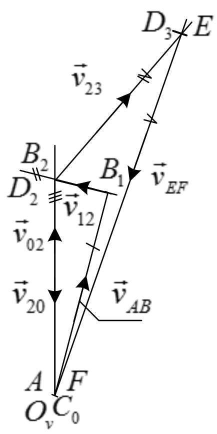
7.2.

Adott: A mechanizmus méretei, pillanatnyi helyzete, és a meghajtás: $\vec{\omega}_{01}$.

Feladat: gyorsulásábra megrajzolása az adott helyzetben.



Sebességábra:



Megoldás:

Szerkezeti képlet: $A\,\overset{\downarrow}{\underline{B}}C \leftarrow DEF$

$$h_g = (1+2+1-3)+(1+1+1-3) = 1+0 = 1$$

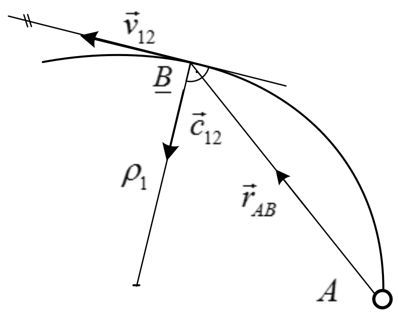
$$h_k = (1-1)+(0-0) = 0+0 = 0$$

Sebességállapot:

1. lánc $\quad \underline{\vec{v}}_{AB} + \underline{\vec{v}}_{12} + \underline{\vec{v}}_{20} = \vec{0}$

2. lánc $\quad \underline{\vec{v}}_{02} + \underline{\vec{v}}_{23} + \underline{\vec{v}}_{EF} = \vec{0}$

Gyorsulási állapot:

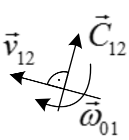


1. lánc: $\quad \vec{a}_{AB} + \vec{a}_{12} + \vec{a}_{20} = \vec{0}$

$$\left(\vec{b}_{AB} + \vec{c}_{AB}\right) + \left(\vec{b}_{12} + \vec{c}_{12} + \vec{C}_{12}\right) + \left(\vec{b}_{02} + \vec{c}_{20} + \vec{C}_{20}\right) = \vec{0},$$

$$\vec{b}_{AB} = \underbrace{\vec{\varepsilon}_{01}}_{\vec{0}} \times \vec{r}_{AB} = \vec{0} \quad \leftarrow \quad \vec{\omega}_{01} = áll., \qquad \vec{c}_{AB} = -\omega_{01}^2 \vec{r}_{AB} \quad \rightarrow \quad \vec{c}_{AB} \,||\, \vec{r}_{AB};$$

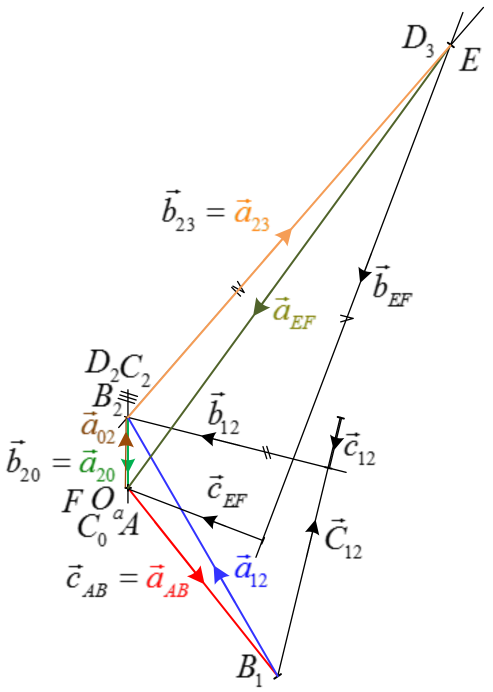
$\vec{b}_{12} \,||\, \vec{v}_{12}$: érintő irányú, $\quad c_{12} = \dfrac{v_{12}^2}{\rho_1}$: normál irányú, $\quad \vec{C}_{12} = 2\vec{\omega}_{01} \times \vec{v}_{12} \quad \Rightarrow \vec{C}_{12} \perp \vec{v}_{12}$;



$\vec{b}_{20}$: csúszka irányú, $\qquad c_{20} = \underset{\infty}{\frac{v_{20}^2}{\rho_2}} = 0, \qquad \vec{C}_{20} = 2\underset{=\vec{0}}{\vec{\omega}_{02}} \times \vec{v}_{20} = \vec{0};$

$$\Big(\underline{\underline{\vec{b}_{AB}}} + \underline{\underline{\vec{c}_{AB}}}\underset{\vec{0}}{}\Big) + \left(\underline{\vec{b}_{12}} + \underline{\underline{\vec{c}_{12}}} + \underline{\underline{\vec{C}_{12}}}\right) + \Big(\underline{\vec{b}_{20}} + \underset{\vec{0}}{\vec{c}_{20}} + \underset{\vec{0}}{\vec{C}_{20}}\Big) = \vec{0},$$

$$\underset{\vec{a}_{AB}}{\underline{\underline{\vec{c}_{AB}}}} + \underbrace{\underline{\underline{\vec{C}_{12}}} + \underline{\underline{\vec{c}_{12}}} + \underline{\vec{b}_{12}}}_{\vec{a}_{12}} + \underset{\vec{a}_{20}}{\underline{\vec{b}_{20}}} = \vec{0} \quad \Rightarrow \quad \left.\begin{matrix}\vec{b}_{12}\\ \vec{b}_{20}\end{matrix}\right\} \quad \Rightarrow \quad B_2, C_2, D_2;$$



$$\vec{a}_{AB} + \vec{a}_{12} + \vec{a}_{20} = \vec{0}$$
$$\vec{a}_{02} + \vec{a}_{23} + \vec{a}_{EF} = \vec{0}$$

2. lánc: $\qquad \vec{a}_{02} + \vec{a}_{23} + \vec{a}_{EF} = \vec{0}$

$$\Big(\underbrace{\vec{b}_{02} + \vec{c}_{02} + \vec{C}_{02}}_{\vec{a}_{02} = -\vec{a}_{20}}\Big) + \left(\vec{b}_{23} + \vec{c}_{23} + \vec{C}_{23}\right) + \left(\vec{b}_{EF} + \vec{c}_{EF}\right) = \vec{0},$$

$\vec{b}_{23} \parallel \vec{v}_{23}$ csúszka irányú (IV),

$c_{23} = \frac{v_{23}^2}{\rho_3} = 0 \quad \rightarrow \quad \vec{v}_{23}$ egyenes vonalú mozgás $\quad \rightarrow \rho_3 = \infty \quad \rightarrow \vec{c}_{23} = \vec{0},$

$\vec{C}_{23} = 2\underset{=\vec{0}}{\vec{\omega}_{02}} \times \vec{v}_{23} = \vec{0};$

$\vec{b}_{EF} = \vec{\varepsilon}_{04} \times \vec{r}_{EF} \quad \rightarrow \vec{b}_{EF} \perp \vec{r}_{EF};$

$\vec{c}_{EF} = -\omega_{04}^2 \vec{r}_{EF} \quad \rightarrow \vec{c}_{EF} \parallel \vec{r}_{EF}, \quad \rightarrow \quad \omega_{04} = \frac{v_{EF}}{r_{EF}} \quad \Rightarrow \quad |c_{EF}|$, (sebességábrából ismert);

$$\underset{\vec{a}_{02}}{-\underline{\underline{\vec{a}_{20}}}} + \underset{\vec{a}_{23}}{\underline{\vec{b}_{23}}} + \underbrace{\underline{\vec{b}_{EF}} + \underline{\underline{\vec{c}_{EF}}}}_{\vec{a}_{EF}} = \vec{0} \quad \Rightarrow \quad \left.\begin{matrix}\vec{b}_{23}\\ \vec{b}_{EF}\end{matrix}\right\} \quad \Rightarrow D_3, E.$$